Green Production through Competitive Testing

Erica L. Plambeck and Terry A. Taylor

Graduate School of Business, Stanford University, Stanford, CA 94305

Graduate School of Business, Columbia University, New York, NY 10027

Abstract

Electronics waste is severely damaging to the environment and human health, especially in developing countries. New regulations in the European Union, California and China prohibit the sale of electronics containing certain hazardous substances. However, because testing for these substances is expensive and destructive of the product, regulators cannot test all or even a significant fraction of the electronics sold. Electronics manufacturers have an incentive to test competitors' products, reveal violations to the regulator, and thus gain market share when the competitors' products are blocked from the market. We find that in many cases, regulators should not test products directly, but instead should rely on electronics manufacturers to do all the testing. Relying on competitive testing is most effective in markets dominated by a few firms and, conversely, is least effective in highly competitive markets composed of many small firms. Unfortunately, in the long run, reliance on competitive testing causes entry and expanded production by manufacturers with low quality, weak brands and consequently low compliance. The phenomenon of competitive testing has the potential to play out in any competitive market governed by product-based environmental, health or safety standards, and our insights apply more broadly to these settings.

Subject Classifications: Cournot oligopoly; environmental regulation; industry self-regulation

January 2007

1 Introduction

Electronics contain heavy metals and other potentially toxic substances, and constitute a fast-growing portion of the waste stream. The United States alone scraps approximately 400 million units per year (Daly 2006), and electronics account for 40% of the lead in U.S. landfills, which threatens groundwater (Eilperin 2005). Used electronics are exported to developing countries and illegally burned to extract valuable metals; the resulting air and water pollution is severely damaging to human health (Basu 2006).

The sage of operations management, W. E. Deming, famously said "build quality in," which implies that to solve the environmental problems with e-waste, hazardous substances should be eliminated from *production*. Espousing this principle, the European Union (EU), China, and California are moving to prohibit the sale of electronics containing six restricted hazardous substances.[1] However, because testing for these restricted substances is complex, expensive and destructive of the product, regulators cannot test all or even a significant fraction of the electronics sold. Instead, regulators may choose to rely upon electronics manufacturers to test their competitors' products. In the United Kingdom, regulators will inspect products in response to "notification of concern from external parties" (EU RoHS 2006). The California department of toxic substances promises to follow up on any violation reported via its website www.dtsc.ca.gov/database/CalEPA_Complaint/index.cfm. When a regulator finds that an electronics manufacturer's product violates the Restrictions on Hazardous Substances (RoHS), the regulator will prevent the sale of that product. For example, Dutch authorities halted the sale of PlayStation consoles because a peripheral cable contained cadmium, which caused Sony to miss $110 million in revenue (Shah and Sullivan 2002). Industry analysts believe that Dutch authorities tested the PlayStation cable for cadmium in response to a tip from one of Sony's competitors (Hess 2006). Electronics manufacturers have an incentive to test competitors' products, reveal violations to the regulator, and thus gain market share when the competitors' products are blocked from the market.

This paper examines whether a regulator should test products directly, or instead rely solely upon manufacturers to test their competitors' products. Further, we examine the impact of competitive testing on the structure, output, profitability and environmental impacts of the electronics industry.

[1] Lead, mercury, cadmium, hexavalent chromium, and two types of brominated flame retardants.

Electronics manufacturers know better than any regulator which components and materials are likely to harbor which restricted substances, and also have a better understanding of the cost of compliance. A relevant economics literature examines pollution prevention when the regulatory agency lacks information about the costs, benefits and/or means of environmental improvement and has incentive problems. For example, Lewis (1996) and references therein show how manufacturers' private information about the cost of reducing emissions may prevent or complicate the implementation of emissions-permit-markets or taxes on emissions. Boyer et al. (2000) point out that the regulator has no incentive to exert monitoring effort when firms are perfectly compliant; this distorts monitoring and investment in environmental improvement from the socially optimal levels. The contribution of this paper is examine the engagement of manufacturers in the testing process, which helps to overcome these problems of asymmetric information.

Recently, other researchers have examined alternative forms of industry self-regulation of environmental impacts. Motivated by the "Responsible Care" program initiated by chemicals manufacturers, Maxwell et al. (2000) model Cournot oligopolists that voluntarily reduce pollution, in order to head off government regulation that would be more stringent and costly. In the tuna industry, nuclear power industry, and others, stakeholders have difficulty discerning the environmental impacts of individual firms and may therefore sanction the entire industry. King et al. (2002) examine strategies (sharing of best practices, standardized reporting, elite clubs, etc.) to cope with this reputational commons problem. Reinhardt (2000) argues that firms with relative advantage in environmental improvement should press for standards and regulation through industry associations, as in the example of the American Forest and Paper Association's Sustainable Forestry Initiative.

The next section introduces our model and concludes by providing a road map for the rest of the paper.

2 Model Formulation

Consider N manufacturers with vertically-differentiated quality. Consumers perceive that manufacturer n's product has quality $u_n > 0$, and are heterogenous in their valuation for quality. Specifically, the market contains a unit mass of consumers with quality valuation parameter v uniformly distributed on $[0, \overline{v}]$. A consumer with quality valuation v who purchases product n at price

p_n has utility

$$u_n v - p_n.$$

Each consumer may purchase one product or nothing, and in the latter case has zero utility. The valuation parameter v may represent differences in income as in Shaked and Sutton (1982) or in taste as in Motta (1993). We index the firms according to their quality:

$$u_1 \leq u_2 \leq ... \leq u_N. \tag{1}$$

Each manufacturer n chooses his production quantity Q_n and his compliance effort e_n to eliminate hazardous substances. Manufacturer n's production cost is $\theta\left[c(e_n)Q_n + F(e_n)\right]$ and all units produced are compliant with RoHS with probability e_n. The fixed cost of compliance $\theta F(e_n)$ arises from product and process development (R&D), component supplier qualification and selection, investment in lead-free soldering equipment, inventory tracking and monitoring systems (IT), and legal and consulting fees. The per-unit production cost $\theta c(e_n)$ reflects the cost of substituting alternative materials for the hazardous substances in various components, yield problems with the new materials, and component inspection. Because a product contains hundreds of subcomponents, and the material in each must be RoHS-compliant in order for the entire product to be RoHS-compliant, manufacturer n cannot guarantee perfect compliance: $e_n \leq \overline{e} < 1$. We assume $c(0) > 0$, $F(0) = 0$, and both $c(e_n)$ and $F(e_n)$ are increasing and strictly convex with $\lim_{e_n \uparrow \overline{e}} \left[c(e_n) + F(e_n)\right] = \infty$ for some $\overline{e} \in (0, 1)$. Throughout the paper, we adopt the convention that "increasing (decreasing)" means "weakly increasing (decreasing)."

The regulator knows the cost functions $c(\cdot)$ and $F(\cdot)$ but has uncertainty about the magnitude of compliance costs, represented by the random variable θ. The regulator knows only the distribution of θ, which has support $[\underline{\theta}, \overline{\theta}]$ where $0 < \underline{\theta} \leq \overline{\theta} < \infty$. In contrast, the firms perfectly understand the cost of compliance. That is, the firms know $c(\cdot)$, $F(\cdot)$, and the realization of θ.

Given the compliance efforts of all N manufacturers, the expected environmental damage is

$$\Sigma_{n=1}^{N} x(1 - e_n)Q_n$$

where x is a positive constant. If the regulator did not impose RoHS, the manufacturers would choose not to incur the extra costs to eliminate hazardous substances, and so environmental damage would be $\Sigma_{n=1}^{N} xQ_n$. The restricted hazardous substances have various environmental impacts that are not fully understood, and assigning a single monetary value to these impacts is very difficult.

One may interpret x as the regulator's expected environmental cost per unit of noncompliant production, or as the regulator's best estimate of society's willingness to pay to eliminate the hazardous substances.

Each manufacturer n also chooses his expenditure t_{nm} in testing the product of his competitor-manufacturer m for hazardous substances, for $n, m \in \mathcal{N} \equiv \{1, .., N\}$ and $m \neq n$. If he finds hazardous substances in the competitor's product, he reports the violation to the regulator. In addition, the regulator (she) spends t_{Rm} in testing the product of manufacturer m for hazardous substances, for $m \in \mathcal{N}$. If the units produced by manufacturer n are noncompliant (recall that this event occurs with probability $1 - e_n$), then testing by the other manufacturers and regulator lead to the detection of the hazardous substances with probability $d\left(\alpha t_{Rn} + \Sigma_{m \in \mathcal{N} \backslash n} t_{mn}\right)$. The probability of detection is strictly increasing and strictly concave with $d(0) = 0$ and $\lim_{t \uparrow \infty} d(t) = \overline{d} \in (0, 1)$. The regulator is less effective than the manufacturers in testing, which is represented by $\alpha \in (0, 1]$.

If hazardous substances are detected in the units produced by manufacturer n, then the regulator will prevent manufacturer n from selling any units in the market. Thus, with probability

$$s\left(e_n, \alpha t_{Rn} + \Sigma_{m \in \mathcal{N} \backslash n} t_{mn}\right) \equiv 1 - d\left(\alpha t_{Rn} + \Sigma_{m \in \mathcal{N} \backslash n} t_{mn}\right)(1 - e_n)$$

manufacturer n successfully brings his full production quantity Q_n to market; with probability $d\left(\alpha t_{Rn} + \Sigma_{m \in \mathcal{N} \backslash n} t_{mn}\right)(1 - e_n)$ manufacturer n is prohibited from doing so. That is, the sales quantity for manufacturer $n \in \mathcal{N}$ is the random variable

$$\widetilde{Q}_n = \begin{cases} Q_n & \text{with probability } s\left(e_n, \alpha t_{Rn} + \Sigma_{m \in \mathcal{N} \backslash n} t_{mn}\right) \\ 0 & \text{with probability } 1 - s\left(e_n, \alpha t_{Rn} + \Sigma_{m \in \mathcal{N} \backslash n} t_{mn}\right). \end{cases} \tag{2}$$

For a given vector of the manufacturers' compliance and testing efforts, we assume that $\widetilde{Q}_n$ and $\widetilde{Q}_m$ for $m \neq n$ are independent.

Under the standard condition $\Sigma_{n=1}^{N} Q_n < 1$, which is necessary to invert the demand functions under Cournot competition with vertically differentiated products (Motta 1993), the market equilibrium price per unit for manufacturer n's product is

$$p_n = \overline{v}(u_n(1 - \Sigma_{m=n}^{N} \widetilde{Q}_m) - \Sigma_{m=1}^{n-1} u_m \widetilde{Q}_m); \tag{3}$$

we refer the reader to (Motta 1993) for the derivation of (3). Then, manufacturer n's expected profit under cost multiplier θ is

$$\pi_n = \overline{v}\left[u_n\left(1 - \Sigma_{m=n+1}^{N} s\left(e_m, \alpha t_{Rm} + \Sigma_{j\in\mathcal{N}\backslash m} t_{jm}\right) Q_m - Q_n\right) - \Sigma_{m=1}^{n-1} u_m\ s\left(e_m, \alpha t_{Rm} + \Sigma_{j\in\mathcal{N}\backslash m} t_{jm}\right) Q_m\right]$$

$$\times\, s\left(e_n, \alpha t_{Rm} + \Sigma_{m\in\mathcal{N}\backslash n} t_{mn}\right) Q_n - \theta[c(e_n)Q_n + F(e_n)] - \Sigma_{m\in\mathcal{N}\backslash n} t_{nm}. \tag{4}$$

If manufacturer n anticipates testing by the regulator of t_{Rk} for $k \in \mathcal{N}$ and production quantity, compliance effort, and testing by the other manufacturers of (Q_m, e_m, t_{mj}) for $m \in \mathcal{N}\backslash n$ and $j \in \mathcal{N}\backslash m$, then manufacturer n chooses his own production, compliance effort and testing (Q_n, e_n, t_{nm}) for $m \in \mathcal{N}\backslash n$ to maximize (4).

Initially, in §3 and §4, we focus on compliance effort e_n and testing t_n decisions; Q_n is fixed at strictly positive level, and this may be interpreted as manufacturer n's capacity. In these sections, when the firms are symmetric (i.e., $u_n = u$ and $Q_n = Q$ for $n = 1, .., N$) we impose the conditions that the detection function is sufficiently concave, the cost functions are increasingly convex, and the marginal cost of an infinitesimally small compliance effort is not too large

$$d''(t) < -\max\left(\frac{d'(t)^2}{d(t)}, \frac{\overline{v}u(N-1)d'(t)^2}{(1-\overline{d})\underline{\theta}[c''(0)Q + F''(0)]}, \underline{d}\right) \text{ for some } \underline{d} > 0 \tag{5}$$

$$c'''(e)Q + F'''(e) \geq 0, \tag{6}$$

$$\overline{\theta}[c'(0)Q + F'(0)] < \overline{v}u(1 - NQ)Q/2. \tag{7}$$

These conditions ensure the existence of a unique symmetric equilibrium (see §3). Similar conditions ensure the existence of an equilibrium in the case with asymmetric firms; our formal results do not require these conditions.

The remainder of the paper is organized as follows. In §3 and §4, we focus on the short-run equilibrium in compliance and testing. In §3, we show that for moderate values of the environmental cost parameter x, the regulator should impose RoHS but rely on the manufacturers to do the testing. Thereafter, we focus on competitive testing ($t_{Rn} = 0$ for $n = 1, .., N$). In §4, we examine analytically the short-run equilibrium in compliance effort and testing. In §5, we present a numerical study of the long-run equilibrium in entry, production quantity, compliance effort and testing. We draw conclusions in §6. All proofs are in the appendix, with the exception that the proof of Proposition 2 is in Plambeck and Taylor (2007).

3 The Role of the Regulator

As noted above, in this section, we take each manufacturer's quantity $Q_n > 0$ as given, and examine the short-run equilibrium in compliance and testing. This is appropriate when the manufacturer's quantity is essentially determined by long-term investments in capacity, and so is fixed over the time horizon in which the firms make compliance and testing decisions. This section focuses on the question of whether the regulator should impose RoHS and whether she should directly test manufacturers. We address this question after first establishing some properties of the equilibrium in compliance and testing. Finally, we describe the impact of RoHS and regulator testing on the firms' expected profits.

In this section, we assume that the regulator can publicly commit to a level of testing expenditure before the manufacturers decide upon their own testing and compliance efforts. We say that the regulator *imposes RoHS* if she responds to noncompliance, detected either by her own testing or by a manufacturer's testing, by preventing the sale of the noncompliant firm's units in the market. We refer to the case in which the regulator imposes RoHS but does not test and instead relies on the manufacturers to test their competitors' products and report violations as *competitive testing*.

Initially, suppose that manufacturers are symmetric: $u_n = u$ and $Q_n = Q$ for $n = 1, .., N$ with $NQ < 1$, so political pressures for fairness compel the regulator to apply the same level of testing to each manufacturer: $t_{Rn} = t_R$ for $n = 1, .., N$. Then manufacturer n's expected profit under cost multiplier θ simplifies to

$$\pi_n = \overline{v}u \left[1 - \Sigma_{m \in \mathcal{N} \backslash n} s\left(e_m, \alpha t_R + \Sigma_{j \in \mathcal{N} \backslash m} t_{jm}\right) Q - Q\right] \; s\left(e_n, \alpha t_R + \Sigma_{m \in \mathcal{N} \backslash n} t_{mn}\right) Q$$

$$- \theta[c(e_n)Q + F(e_n)] - \Sigma_{m \in \mathcal{N} \backslash n} t_{nm}.$$

Because the firms are symmetric, it is natural to focus on symmetric equilibria. Lemma 1 establishes the existence of a unique symmetric equilibrium and describes some of its characteristics. Hereafter we assume that the symmetric firms play this equilibrium.

Let $(\widehat{e}(\theta, t_R), \widehat{t}(\theta, t_R))$ denote the symmetric equilibrium in compliance and testing with cost multiplier θ and regulator's testing expenditure per firm t_R. When a manufacturer applies $\widehat{t}(\theta, t_R)$ to each of his $N-1$ competitors, the manufacturer's total testing expenditure is $\widehat{T}(\theta, t_R) = (N-1)\widehat{t}(\theta, t_R)$, and this $\widehat{T}(\theta, t_R)$ is also the total of the manufacturers' testing expenditures applied to each firm. Henceforth, we describe the symmetric equilibrium in terms of $\widehat{T}(\theta, t_R)$.

Lemma 1 *Suppose the firms are symmetric. Under RoHS, for any testing level by the regulator* $t_R \geq 0$ *and any realized cost multiplier* θ*, there exists a unique symmetric equilibrium in compliance and testing. If the regulator's testing expenditure is small* $t_R \leq \widehat{T}(\theta, 0)/\alpha$*, then testing by the regulator has no effect on the equilibrium compliance and detection probability; in the unique symmetric equilibrium,*

$$\widehat{e}(\theta, t_R) = \widehat{e}(\theta, 0)$$

$$d(\alpha t_R + \widehat{T}(\theta, t_R)) = d(\widehat{T}(\theta, 0)).$$

Otherwise, if the regulator's testing expenditure is large $t_R > \widehat{T}(\theta, 0)/\alpha$*, then in the unique symmetric equilibrium the firms do not test:*

$$\widehat{T}(\theta, t_R) = 0,$$

and testing by the regulator results in greater compliance and a strictly higher detection probability:

$$\widehat{e}(\theta, t_R) \geq \widehat{e}(\theta, 0) \tag{8}$$

$$d(\alpha t_R + \widehat{T}(\theta, t_R)) > d(\widehat{T}(\theta, 0)).$$

For the regulator, spending a small amount on testing ($t_R \leq \widehat{T}(\theta, 0)/\alpha$) is completely ineffective in influencing compliance or detection. At any such testing expenditure by the regulator, the firms' marginal value for testing is positive; consequently, the firms will test so as to bring the detection probability for each firm up to its level without regulator involvement $d(\widehat{T}(\theta, 0))$, leaving the incentives for compliance unchanged. In contrast, high testing expenditures by the regulator ($t_R > \widehat{T}(\theta, 0)/\alpha$), by increasing the probability that a firm's noncompliance will be detected, provide stronger incentives for compliance. At any such testing expenditure by the regulator, the firms' marginal value for testing is negative, and consequently the firms do not test.

We now turn to the central question: should the regulator impose RoHS and if so, should she directly test manufacturers' products for RoHS-compliance? We say that the regulator should impose RoHS if and only if this strictly increases expected social welfare, and should test manufacturers products ($t_R > 0$) if and only if this strictly increases expected social welfare. Expected social welfare under RoHS is the utility the units create less the cost of production, environmental damage, and testing

$$E_\theta \left[\overline{v}u \Sigma_{n=1}^{N} \widetilde{Q}_n (1 - \Sigma_{n=1}^{N} \widetilde{Q}_n/2) - N \left[\theta [c(\widehat{e}(\theta, t_R))Q + F(\widehat{e}(\theta, t_R))] + x(1 - \widehat{e}(\theta, t_R))Q + \widehat{T}(\theta, t_R) \right] \right] - N t_R. \tag{9}$$

Note that $\widehat{e}$ and $\widehat{T}$, which depend on t_R, determine the distribution of $\widetilde{Q}_n$ as specified in (2). Without RoHS, anticipating no penalty from noncompliance, the firms do not invest in compliance, and expected social welfare simplifies to

$$\overline{v}u \; NQ(1 - NQ/2) - N(E[\theta]c(0) + x)Q.$$

Our main result is that for moderate levels of the environmental cost parameter x, the regulator should impose RoHS and rely on competitive testing.

Theorem 1 *There exist thresholds*

$$0 \leq \underline{x} \leq \overline{x} \tag{10}$$

such that the regulator should impose RoHS if and only if $x > \underline{x}$ and should test if and only $x > \overline{x}$. Sufficient but not necessary conditions for

$$\underline{x} < \overline{x}$$

are that $\alpha < \overline{\alpha}$ for some $\overline{\alpha} \in (0, 1]$ and $\Pr(\widehat{e}(\theta, 0) > 0) > 0$.

Imposing RoHS positively impacts social welfare by reducing environmental damage, but negatively impacts social welfare by imposing testing costs, increasing production costs, and keeping goods out of the hands of consumers. Testing by the regulator amplifies both the positive and negative impacts. When the environmental cost parameter is small $x \leq \underline{x}$, the negative impacts of imposing RoHS outweigh the positive impact from greater compliance. When the environmental cost parameter is large $x > \overline{x}$, to provide strong incentives for compliance the regulator should impose RoHS and directly test the manufacturers. When the environmental cost parameter is moderate, $x \in (\underline{x}, \overline{x}]$, the regulator should impose RoHS and rely on competitive testing. The condition $\Pr(\widehat{e}(\theta, 0) > 0) > 0$ means that with positive probability, competitive testing induces some compliance effort. If this condition were violated, a regulator would never impose RoHS and rely solely on competitive testing. We are very confident that $\Pr(\widehat{e}(\theta, 0) > 0) > 0$ is satisfied in practice. When the regulator's testing efficacy is low $\alpha < \overline{\alpha}$, testing by the regulator is socially inefficient in that the regulator's inefficient testing displaces the more efficient testing the manufacturers would exert under competitive testing. This favors competitive testing over regulator testing, which explains why the parameter region $(\underline{x}, \overline{x}]$ is nonempty.

These insights are best illustrated and extended with a numerical study. Throughout the paper, in our numerical studies we focus on the functional forms $F(e) = \overline{F}(-\log(\overline{e}-e)-e)$, $c(e) = \underline{c}$ and $d(t) = \overline{d}(\sqrt{a^2t^2+4at} - at)/2$. We assume that

$$\theta = \begin{cases} 1-\Delta & \text{with probability } 1/2 \\ 1+\Delta & \text{with probability } 1/2. \end{cases}$$

The detection function $d(t)$ is increasing in a, and a can be interpreted as a measure of testing efficacy. Parameters are $\alpha = 1$, $\overline{v} = 100$, $u = 1$, $\overline{e} = \overline{d} = 0.99$, $\underline{c} = 40$, and all possible combinations of $\Delta = \{0, 0.1, 0.2, .., 0.9\}$, $\overline{F} = \{0.1, 0.5, 1\}$, $a = \{1, 10, 20\}$ and $N = \{2, 5, 8\}$. For each experiment with $N = 2$, 5, and 8, we set $Q = 0.2$, 0.1 and 0.07, respectively. These are the equilibrium optimal levels of Q for the firms in the median case that $\theta = E[\theta] = 1$, $\overline{F} = 0.5$, and $a = 10$ (see Figure 2 in Section 5).

From Theorem 1, there exists a range of values of the environmental cost parameter for which the regulator should impose RoHS and rely on competitive testing, provided that the inequality $\underline{x} < \overline{x}$ is strict. We observed strict inequality $\underline{x} < \overline{x}$ in almost all (93%) of the parameter settings.[2] Across all parameter settings, the ratio of $\overline{x}/\underline{x}$ had median 2.4 and mean 2.5. The maximum $\overline{x}/\underline{x} = 50$ was attained at the parameter setting with the highest testing efficacy ($a = 20$), lowest cost ($\overline{F} = 0.1$), least competition ($N = 2$) and highest uncertainty for the regulator ($\Delta = 0.9$). For this parameter setting, the equilibrium detection probability in the absence of testing by the regulator was reasonably high: 0.29 in the event $\theta = 0.1$ and 0.5 in the event $\theta = 1.9$. An increase in a increases the competitive equilibrium detection probability and hence compliance, which tends to make testing by the regulator unnecessary. Similarly, a decrease in the number of firms N increases the competitive equilibrium compliance and thus tends to make testing by the regulator unnecessary. The maximal $\overline{x}/\underline{x}$ occurs with minimum N and, conversely, $\underline{x} = \overline{x}$ occurs only with the maximal N.

The large magnitude of the gap between $\underline{x}$ and $\overline{x}$ in our numerical is surprising, given our assumption that the regulator is just as effective as the manufacturers in testing, i.e., $\alpha = 1$. Most surprising is that $\underline{x} < \overline{x}$ when $\Delta = 0$, meaning that the regulator also perfectly knows the cost

[2] We observe $\underline{x} = \overline{x}$ only in cases with low testing efficacy a, high cost $\overline{F}$, high competition (large N), and medium to high levels of the regulator's uncertainty about cost Δ. In these cases, the equilibrium detection probability in the absence of regulatory testing is extremely low. Therefore, if the environmental cost is sufficiently high to justify imposing RoHS, then testing by the regulator is necessary to provide stronger incentives for compliance.

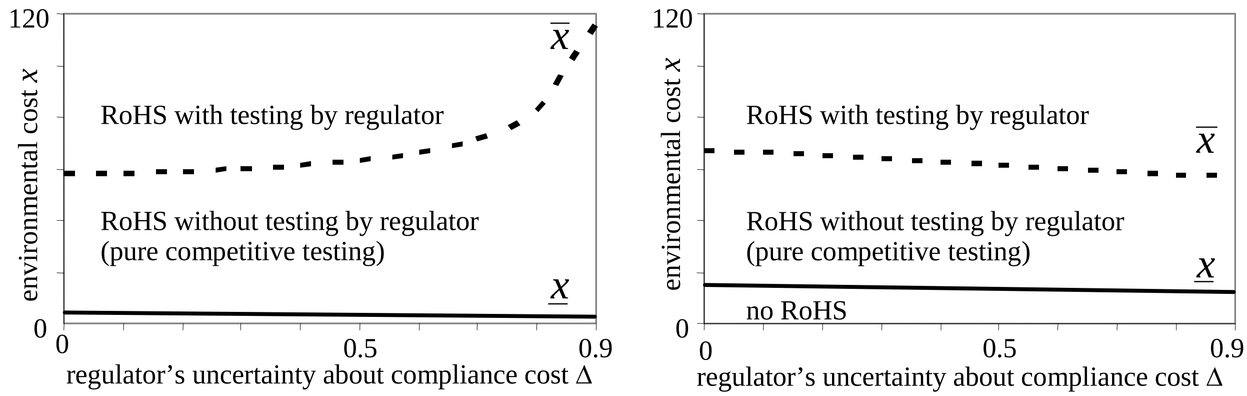

Figure 1: Environmental cost thresholds for the regulator to impose RoHS $\underline{x}$ and to impose RoHS *and* test products for compliance $\overline{x}$, as a function of the regulator's uncertainty about the manufacturer's cost of compliance. In both panels $N = 2$ and $a = 20$. In the left panel $\overline{F} = 0.1$ and in the right panel $\overline{F} = 1$.

of compliance. The explanation is that under RoHS with competitive testing, for low levels of environmental cost x, the manufacturers exert more testing effort (and knock more products out of the market) than is optimal for social welfare, so additional testing by the regulator becomes desirable only at strictly higher levels of the environmental cost parameter. In reality, manufacturers are much better than the regulator at detecting noncompliance ($\alpha \ll 1$). Because $\underline{x}$ is invariant with α but $\overline{x}$ increases sharply with α, our numerical study suggests that the regulator should not test, but rely on competitive testing (at least when the number of firms is not too large).

This conclusion is reinforced by the following counterintuitive but very robust result. For all parameter settings, $\underline{x}$ strictly decreases with Δ. That is, the more uncertain the regulator is about the cost of compliance, the more readily she should impose RoHS. (Indeed, in reality, the regulator is very uncertain about the cost of compliance.) This counterintuitive result occurs because the manufacturers adapt their testing and compliance efforts to the realization of the cost multiplier θ, so that compliance is high (low) when θ is low (high), which becomes increasingly advantageous as we hold $E[\theta] = 1$ and increase the high and low realizations of θ. In contrast, Figure 1 shows that the threshold for regulatory testing $\overline{x}$ may increase or decrease with the regulator's uncertainty about the cost of compliance Δ. For all parameter settings, we observed that $\underline{x}$ increases with the compliance cost multiplier $\overline{F}$ and, if the regulator knows the compliance cost ($\Delta = 0$), then $\overline{x}$ also increases with the compliance cost multiplier $\overline{F}$. (This phenomenon is illustrated in Figure 1; note that $\overline{F}$ is 0.1 in the left panel and 1 in the right panel.) However, when the regulator is uncertain about the compliance cost ($\Delta > 0$), $\overline{x}$ may increase or decrease with the compliance cost multiplier $\overline{F}$. Further, both thresholds $\underline{x}$ and $\overline{x}$ may increase or decrease with the testing efficacy a.

We now turn to the impact of regulation on the profitability of the firms. One might conjecture that a firm would prefer that the regulator not impose RoHS and not test, because such actions increase the likelihood that the firm's products will be blocked from the market. This conjecture is false: in some cases all the firms prefer that the regulator imposes RoHS (with or without regulator testing) instead of not imposing RoHS. The intuition is that imposing RoHS increases the expected price (by limiting the selling quantity) and testing by the regulator saves the firms the cost of testing their competitors. For example, with symmetric firms and parameters $\alpha = 1$, $\overline{v} = 100$, $u = 1$, $\overline{e} = \overline{d} = 0.99$, $\Delta = 0$, $\underline{c} = 40$, $\overline{F} = 5$, $a = 10$, $N = 2$, $Q = 0.45$ and $x \in (28, 71)$, each firm's expected profit is 4.4 without RoHS, is 4.9 under RoHS, and is 5.4 under RoHS with regulator testing $t_R = 0.6$. In this example $\underline{x} = 28$ and $\overline{x} = 71$, so both the firms and society are better off as a result of implementing competitive testing. However, the fact that the firms would be even better off under regulator testing $t_R = 0.6$, suggests that, contrary to conventional wisdom, the firms would lobby for aggressive regulatory testing of their products. Regulators should be wary of such calls for more aggressive regulation, because, as this example (with $x < \overline{x}$) illustrates, it may be detrimental to social welfare.

We have assumed that the regulator can publicly commit to a level of testing expenditure before the firms make their compliance decisions. However, after the firms have made their compliance decisions, the associated environmental costs are "sunk" (invariant with respect to the regulator's testing expenditure). At that point in time, testing by the regulator can only reduce expected social welfare by causing products to be withheld from the market. Therefore, as in Boyer et al. (2000), the regulator might be unable to commit to testing.

In reality, the regulator has great uncertainty about the cost of compliance and is less effective than manufacturers in testing. Therefore, even if the regulator could commit to a positive testing expenditure, Theorem 1 and our numerical study suggest that she should not do so, but instead rely on competitive testing (especially when the market is dominated by a few firms, as is the case in important segments of the electronics industry such as servers, personal computers, and video game consoles). We will henceforth assume that the regulator imposes RoHS but relies on competitive testing. Moreover, because all the manufacturers know the realization of the cost multiplier θ, we will assume without loss of generality that $\theta = 1$. We will write the symmetric equilibrium in compliance and testing as $(\widehat{e}, \widehat{T})$, suppressing the dependency on $\theta = 1$ and $t_R = 0$.

4 Short-Run Equilibrium with Competitive Testing

In this section, we continue to take each manufacturer's quantity $Q_n > 0$ as given, and focus on compliance and testing decisions. As before, we initially focus on the case in which the manufacturers are symmetric; this allows us to perform comparative statics for the unique symmetric equilibrium.

Proposition 1 characterizes the impact of the number of firms and their quality on equilibrium investment levels. Recall that $\widehat{T}$ represents the total equilibrium testing effort both exerted by each firm and applied to each firm.

Proposition 1 *The equilibrium compliance effort $\widehat{e}$ is decreasing in N and increasing in u. There exists $\overline{N}$ such that the equilibrium testing effort $\widehat{T}$ is increasing in N for $N \leq \overline{N}$ and decreasing in N for $N \geq \overline{N}$.*

We begin by explaining the impact of increasing the number of firms N on the total equilibrium testing effort $\widehat{T}$, when the number of firms $N \geq \overline{N}$; although we describe this as the case where the number of firms is "large," for some parameters $\overline{N} = 2$, making this case exhaustive. The total equilibrium testing effort $\widehat{T}$ decreases as the number of firms increases for two reasons. First, as the number of firms increases, each individual competitor has a smaller impact on the overall market and so the value of knocking that competitor out of the market decreases. Second, there is a free rider problem: when one competitor is knocked out of the market, all the remaining firms benefit, and this positive externality causes each firm to underinvest in testing. As the number of firms increases, this free rider problem is exacerbated, weakening the incentive for each firm to test its competitors.

As the number of firms increases, the equilibrium compliance level decreases. The rationale when the number of firms is large $N \geq \overline{N}$ is twofold. First, because as the number of firms increases, less testing effort is applied to each firm, so each firm has a smaller chance of being detected for noncompliance, and consequently has a less incentive to invest in compliance. Second, as the number of firms increases, the market becomes more competitive, decreasing the value of bringing products to market. Because the payoff from compliant production is smaller, each firm has less incentive to invest in compliance. When the number of firms is small $N \leq \overline{N}$, only the second rationale explains why compliance is decreasing in N.

When the number of firms is very small $N \ll \overline{N}$, the level of compliance can be quite high.

Consequently, investments in testing to detect noncompliance tend to be ineffective. As N increases on $N \leq \overline{N}$, the level of compliance decreases, which makes testing investments more likely to pay off, and consequently, the equilibrium testing effort $\widehat{T}$ increases.

Intuitively, as the firms' quality levels increase so that customer willingness to pay increases, the firms have more to lose from being discovered as noncompliant; consequently, the firms invest more in compliance. In our numerical studies we observed that as the quality level increases, the total equilibrium testing effort $\widehat{T}$ also increases.

We conclude that in industries with many manufacturers, each with weak brands, compliance under competitive testing will be low, with consequent environmental costs. This helps explain our numerical observation in §3 that relying on competitive testing is inadequate in terms of social-welfare maximization when the number of firms is large.

Next, we consider asymmetric firms. Under competitive testing, manufacturer n's expected profit simplifies from (4) to

$$\pi_n = \overline{v}\left[u_n\left(1 - \Sigma_{m=n+1}^{N} s\left(e_m, \Sigma_{j\in\mathcal{N}\setminus m} t_{jm}\right) Q_m - Q_n\right) - \Sigma_{m=1}^{n-1} u_m s\left(e_m, \Sigma_{j\in\mathcal{N}\setminus m} t_{jm}\right) Q_m\right]$$

$$\times\, s\left(e_n, \Sigma_{m\in\mathcal{N}\setminus n} t_{mn}\right) Q_n - c(e_n) Q_n - F(e_n) - \Sigma_{m\in\mathcal{N}\setminus n} t_{nm}.$$

By inspection of π_n, we have the following important observations. Manufacturer n's incentive for compliance increases with his own production quantity Q_n and quality u_n, and increases with the other manufacturers' testing of his products $\Sigma_{m\in\mathcal{N}\setminus n} t_{mn}$. Furthermore, every other manufacturer m's incentive to test manufacturer n's products increases with manufacturer m's production quantity Q_m and quality u_m. A common observation in the literature on Cournot oligopoly with vertically differentiated quality is that each firm's production quantity increases with its quality. This suggests that manufacturers with relatively strong brands will draw more testing from their competitors and have higher compliance in equilibrium, whereas manufacturers with weak brands will draw less testing from their competitors and have lower compliance in equilibrium.

The next proposition establishes that if a manufacturer has sufficiently low quality, he does not comply with RoHS, does not test competitors products, and draws less testing from his competitors than other manufacturers with higher quality. Most importantly, that manufacturer with low quality has strictly greater expected profit as a result of the RoHS regulation. The proposition requires two technical assumptions: that the marginal cost of compliance is strictly positive, and small investments in testing yield significantly large detection probabilities.

Proposition 2 *Suppose that $c'(0) + F'(0) > 0$ and $\lim_{t\downarrow 0} d'(t) = \infty$. There exists $u_L > 0$ such that if $u_l \leq u_L$ for $l \in \{1, .., N_L\}$ and $N \geq N_L + 2$, then in any Nash equilibrium, for any $l \in \{1, .., N_L\}$ and $h \in \{N_L + 1, .., N\}$, manufacturer l does not comply, does not test the products of manufacturer h, and draws less testing from its competitors than manufacturer h :*

$$e_l = 0 \tag{11}$$

$$t_{lh} = 0 \tag{12}$$

$$\sum_{j\in\mathcal{N}\backslash l} t_{jl} < \sum_{j\in\mathcal{N}\backslash h} t_{jh}. \tag{13}$$

Furthermore, RoHS strictly increases manufacturer l's expected profit.

In the short run, where the number of firms is fixed, Theorem 1 establishes that imposing RoHS with only competitive testing is socially optimal, provided that the environmental cost of noncompliance x is moderately high. However, Proposition 2 suggests that reliance on competitive testing may have undesirable effects in the longer run. Specifically, Proposition 2's result that RoHS increases expected profit for manufacturers with low perceived quality u implies that this form of regulation increases the incentive for entry by "white-box" manufacturers with weak brands. This is threatening to the environment, especially in light of Proposition 1, which points out that an increase in the number of firms in the market N and/or a reduction in perceived quality u results in lower compliance and greater environmental impacts in equilibrium.

We next examine the effect of RoHS on industry structure in the long run, as firms make entry and production decisions in addition to compliance and testing decisions.

5 Long-run Equilibrium under Competitive Testing

In this section we expand our study to include entry and production quantity decisions, which corresponds to considering the longer-run problem that the firms face. We extend the sequence of events described in §3 so that in the first stage, potential entrants decide whether or not to enter, where entry entails incurring a fixed cost K and allows the entrant to subsequently produce units of quality u. In the second stage, each entrant n observes the number of entering firms N, and then privately invests in production Q_n, compliance e_n and testing t_{nm} for $m \in \mathcal{N}\backslash n$. Although each firm makes these investments sequentially, because these investments are private, from the standpoint

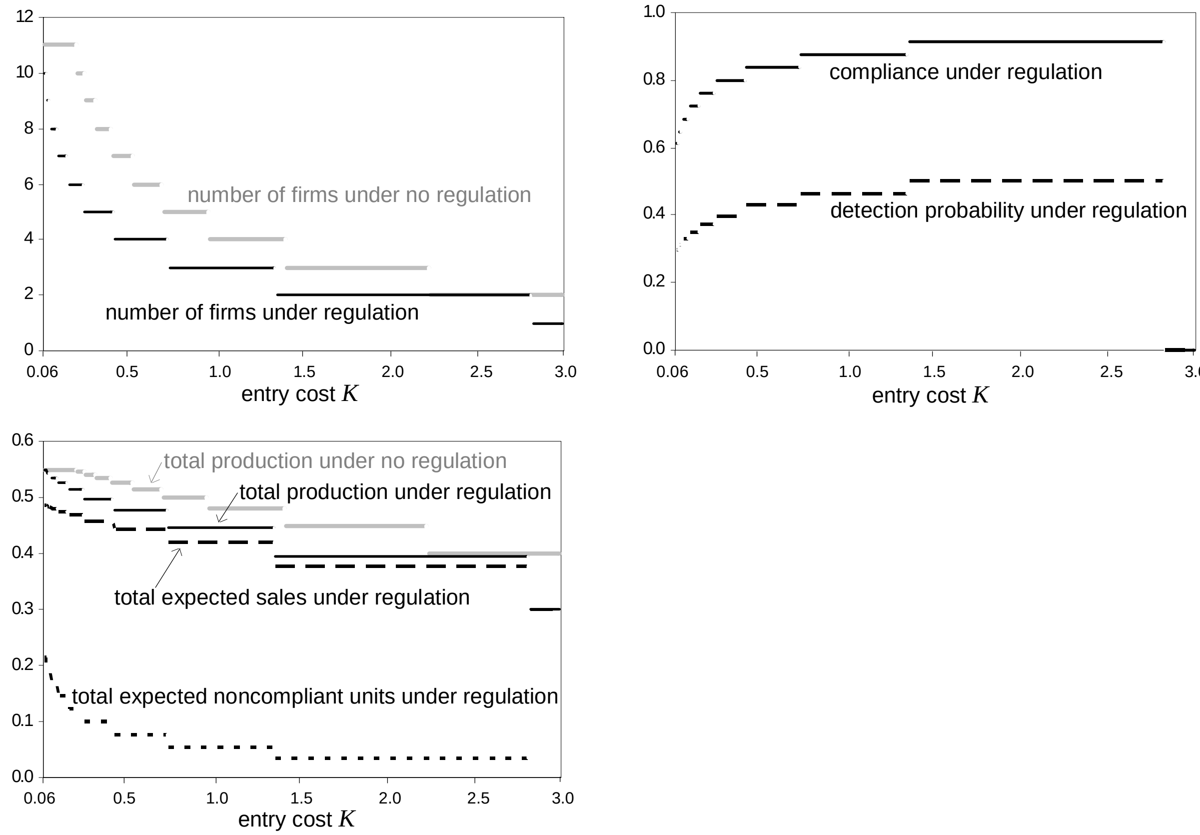

Figure 2: Impact of Entry Cost on Number of Firms, Compliance, Testing, and Quantities.

of characterizing equilibria, it is as if all firms make the three decisions simultaneously. Potential entrants enter in the first stage if and only if they anticipate that their expected profit in the second stage will (weakly) exceed the cost of entry K. Because of the additional complexity introduced by the entry and production decisions, this section presents numerical results. In each instance of our numerical study, regardless of the number of firms that enter, there is a unique symmetric equilibrium in the Stage Two game, and we assume that the firms play this equilibrium.[3]

Figure 2 demonstrates how the firms' entry, compliance and testing decisions depend on the cost of entry. Parameters are $\overline{v} = 100$, $u = 1$, $\overline{e} = \overline{d} = 0.99$, $a = 10$, $\underline{c} = 40$, $\overline{F} = 0.5$, and $K \in [0.06, 3.00]$. Figure 2 is representative of a larger numerical study in which we considered $\overline{v} = 100$, $u = 1$, $\overline{e} = \overline{d} = 0.99$, all parameter combinations of $a = \{1, 10, 20\}$, $\underline{c} = \{10, 20, 40\}$, $\overline{F} = \{0.1, 0.5, 1\}$, and the range of K corresponding to $N \in \{1, 2, .., 11\}$ entrants.

[3] An asymmetric equilibrium might achieve greater social welfare than the symmetric equilibrium when x is large and the cost of compliance is invariant with the production quantity ($c(e) = c$). A firm with relatively low compliance produces relatively little in an asymmetric equilibrium.

Figure 2's top left panel depicts the equilibrium number of firms as a function of the entry cost. In the long-run equilibrium, regulation makes the industry less profitable because each firm faces the prospect that its own products might be withheld from the market, and incurs costs for compliance and for testing its competitors. Consequently, regulation reduces the equilibrium number of firms. As the cost of entry increases, the industry becomes less attractive and fewer firms enter.

Figure 2's top right panel depicts, in the setting with regulation, the compliance e and total industry testing investment per firm T, where the later is measured by the resulting detection probability $d(T)$, as a function of the cost of entry. As the entry cost increases, so that fewer firms enter, the market becomes more attractive to any individual firm, which strengthens the incentive to invest in compliance. Further, with fewer firms, testing expenditure has a larger payoff because the effect of blocking the sale of a competitor's product is more pronounced; this more intensive testing reinforces the incentive to invest in compliance. Consequently, compliance and testing increase in the cost of entry when the cost of entry is not too high. However, when the cost of entry is very high, so that the industry can only support a single firm, then compliance drops to zero because no competing firm tests the monopolist. Similarly, in the setting without regulation, regardless of the entry cost, firms have no incentive for compliance or testing and do not invest in either: $e = T = 0$.

Figure 2's bottom panel depicts the total production NQ under regulation and under no regulation. Under no regulation, this coincides with the total expected sales quantity and total expected noncompliant units because no units are tested and all units produced are noncompliant. However, under regulation, in expectation, only a fraction of total production is converted into sales $s(e,T)NQ$ and only a fraction of the total production is noncompliant $(1-e)NQ$; these quantities are also depicted in the bottom panel. The intuition from the standard Cournot model without regulation carries over to the setting with regulation: as the entry cost increases, so that fewer firms enter, quantity competition becomes less intense, reducing the total production and expected sales quantity. The total expected noncompliant units produced under regulation $(1-e)NQ$ is decreasing in the entry cost, and this occurs for two reasons: Having fewer firms *intensifies competition in testing and compliance* (e is larger) at the same time that it *weakens competition in quantity* (NQ is smaller). The caveat is that as the entry cost increases to level where the industry can only support a single firm, the competition in testing and compliance disappears, so that to the total expected noncompliant quantity jumps up.

In the example above, the manufacturer's compliance cost is due to fixed costs ($F(e)$) rather than variable costs ($c(e)$). If, in addition, compliance increases the cost per unit of production $c(e)$, each firm will produce less and have lower compliance in equilibrium. This decreases the expected sales quantity of competitors, and thus may increase entry despite the adverse increase in production cost.

Our numerical study reinforces the analytical results in the previous section. Namely, these results suggest that in industries with high costs of entry (and correspondingly few firms), competitive testing can be effective in limiting noncompliant production. However, with low cost of entry (and correspondingly many firms), competitive testing will be ineffective. Thus, we obtain the somewhat paradoxical conclusion that in settings that are commonly thought of as being highly competitive, competitive testing fails; it only succeeds in sharply limiting noncompliant production in settings which are less competitive from a product standpoint. The intuition is that highly competitive markets are diffuse and this diffusion undermines the incentive of any individual firm to test its many small competitors.

6 Discussion

Our short-run analysis suggest that in the segments of the electronics industry dominated by a small number of manufacturers with strong brands (e.g., video game consoles), relying on manufacturers to test their competitors' products is effective in encouraging RoHS-compliance. However, in highly competitive consumer electronics markets with many manufacturers we anticipate that the threat of competitive testing will have little positive effect on encouraging compliance.

In the long run, relying on the manufacturers for competitive testing may cause entry by manufacturers with relatively weak brands, and consequently low RoHS-compliance. Even for incumbent manufacturers with strong brands, this increase in competition weakens the incentive for RoHS-compliance. To reduce the incentive for entry and thus increase long-run RoHS compliance, regulators can follow the state of Maine in requiring each brand to register and pay a fixed fee. (In Maine, those fees help to pay for the recovery and recycling of used electronics.) As demonstrated in the numerical study in §5, the total expected noncompliant quantity produced under regulation is decreasing in the entry cost. By increasing the cost of entry, a registration fee reduces the negative environmental impact of noncompliant production. Unfortunately, it also mitigates quantity competition, driving up the expected selling price and making the product available to fewer consumers. The socially optimal fee to register a brand balances these two concerns, with the optimal fee increasing in the environmental cost of noncompliant production.

Another long-run impediment to competitive testing is that manufacturers may develop agreements not to test each others' products, enforced by the explicit or implicit threat "if you report my RoHS-violation now, then in future I will test your products and report any RoHS-violation." Therefore allowing *anonymous* reporting of violations, as does the California Department of Toxic Substances, will encourage competitive testing.

In the long run, manufacturers have a stronger incentive for RoHS compliance than captured in our one-period model in that if a RoHS-violation is detected, the manufacturer's brand and hence future profits will be damaged. This incentive is presumably strongest for manufacturers with a strong reputation and brand, and therefore reinforces our finding that manufacturers with stronger brands have higher RoHS compliance in equilibrium.

In practice, environmental nonprofit organizations are testing big-brand electronics manufacturer's products for RoHS-compliance. Donations to an environmental nonprofit increase with the nonprofit's reputation for efficacy, and hence with positive press coverage. Detection of a RoHS-violation by a big-brand manufacturer will generate much more press coverage than would detection of a RoHS-violation by a small, little-known manufacturer. (For example, Graham-Rowe (2006) covers the detection by Greenpeace of brominated flame retardants in a Hewlett-Packard (HP) computer; the computer was not illegal because it was sold in the EU shortly before the RoHS regulation came into effect, but HP had previously advertised that its products were free of such hazardous substances.) Therefore environmental nonprofit organizations have relatively little incentive to test the products of little-known manufacturers. Testing by nonprofit organizations reinforces our conclusion that manufacturers with stronger brands will have higher RoHS-compliance in equilibrium.

In our model, environment cost is proportional to the quantity of production with restricted hazardous substances, but in reality the environmental cost structure is more complex. Even RoHS-compliant production causes environmental impacts. In particular, U.S. Environmental Protection Agency (EPA) administrators are concerned that manufacturers' substitutes for restricted substances might also be toxic (Lindsay 2006). Moreover, for either a RoHS-compliant product or a non-compliant product, the environmental cost depends upon how the product is treated at the end of its useful life. In the EU, the Waste Electrical and Electronic Equipment (WEEE) directive requires that manufacturers collect and recycle a fraction (50-80% depending on product category) of their products. Recycling reduces some environmental impacts and increases others, but presumably reduces the net environmental cost (Huisman et al. 2003, Mayers et al. 2005). The most fundamental issue is that assigning a dollar value to the environmental impacts of RoHS-compliant versus noncompliant production under various recycling scenarios is very difficult, and requires further research.

Further research is also needed on integrated optimal design of regulation for electronics production and end-of-life management. To maximize expected social welfare, the regulator's budget for RoHS testing and the target fraction of products recycled must be jointly optimized, because RoHS-compliance reduces the net cost of recycling[4] and recycling reduces the environmental cost of noncompliant products and, to lesser extent, RoHS-compliant products. In a model without explicit RoHS, Atasu et al. (2006) characterize the socially optimal fraction of products to recycle. They assume a fixed environmental cost per unit production that is not recycled and zero environmental cost for recycled units. They show that the optimal fraction to recycle increases with the number of competing manufacturers. We have shown that the prevalence of hazardous substances increases with the number of competing manufacturers, which reinforces the need for safe recycling and/or disposal of electronics at end of life.

The U.S. EPA and nonprofit Green Electronics Council have established a website www.epeat.com where electronics manufacturers may voluntarily rank their products as "Gold," "Silver" or "Bronze" based on RoHS compliance, energy efficiency, and other environmental attributes. This system was originally intended to guide U.S. state and federal government procurement, but is also influencing purchases by corporate, nonprofit, and even some individual consumers (Rehfeld 2006). In this voluntary system, manufacturers' incentive to rank products truthfully is the threat of damage to reputation and brand in the event that a violation is detected. Further research is needed to assess how such voluntary systems affect the structure, output, profitability and environmental impacts of the electronics industry.

We conclude by noting that although our model is motivated by the specifics of environmental regulation in the electronics industry, the phenomenon of competitive testing has the potential to play out in any competitive market governed by product-based environmental, health, or safety standards, and our insights apply more broadly to these settings.

[4]For example, elimination of brominated flame retardants allows for plastics to be recycled or safely burned for energy recovery.

References

Atasu, A., M. Sarvary, L. Van Wassenhove. 2006. Efficient take-back legislation. Working Paper, INSEAD, France.

Basu, I. 2006. India, the digital dumping ground *South Asia*, August 3.

Boyer, M., T. R. Lewis, W. L. Liu. 2000. Setting standards for credible compliance and law enforcement. Canadian Journal of Economics **33**(2) 319-340.

Daly, L. 2006. Recycling technology products: An overview of E-waste policy issues. Report by U.S. Department of Commerce, Technology Administration, Office of Technology Policy.

Eilperin, J. 2005. Millions of tons of used devices pose threat to environment. *Washington Post* Jan. 21, A04.

EU RoHS Enforcement Authorities Informal Network. 2006. RoHS Enforcement Guidance Document Version 1, May 1.

Graham-Rowe, D. 2006. Uncovering the hazards in our electronic gadgets. *New Scientist* **191**(2570), September 23, 26-27.

Hess, R. 2006. Ensuring compliance: A content perspective. Presentation at Stanford Global Supply Chain Management Forum, Stanford University, Apr. 19.

Huisman, J., C. Box, A. Stevels. 2003. Quotes for environmentally weighted recyclability. *International Journal of Production Research.* **41**(16) 3649- 3365.

King, A. A., M. J. Lenox, M. L. Barnett. 2002. Strategic responses to the reputation commons problem, in *Organizations, Policy and the Natural Environment*, A. J. Hoffman, M. J. Ventresca, eds. Stanford University Press, Stanford, CA, 393-407.

Lewis, T. R. 1996. Protecting the environment when costs and benefits are privately known. *RAND Journal of Economics* **27**(4) 819-847.

Lindsay, C. 2006. Interview with Administrator for United States Environmental Protection Agency, Office of Solid Waste, Nov. 9.

Maxwell, J. W., T. P. Lyon, S. C. Hackett. 2000. Self-regulation and social welfare: The political economy of corporate environmentalism. *Journal of Law and Economics* **43**(2) 583-617.

Mayers, C. K., C. M. France, S. J. Cowell. 2005. Extended producer responsibility for waste electronics. *Journal of Industrial Ecology* **9**(3) 169-189.

Motta, M. 1993. Endogenous quality choice: Price vs. quantity competition. *Journal of Industrial Economics* **41** 113-131.

Plambeck, E. L., T. A. Taylor. 2007. Green Production through Competitive Testing: Technical Appendix. Working Paper, Stanford University.

Rehfield, B. 2006. When "refurbished" takes on an Earth-friendly vibe. *New York Times* December 31, 6.

Reinhardt, F. L. 2000. Managing competitors, in *Down to Earth: Applying Business Principles to Environmental Management*. Harvard Business School Press, Cambridge, MA 45-78.

Shah, J. B., L. Sullivan. 2002. Firms warned to brace for environmental fallout – New laws threaten to disrupt the unprepared. *Electronic Buyer's News* Sept. 30, 1.

Shaked, A., J. Sutton. 1982. Relaxing product competition through product differentiation. *Review of Economic Studies* **44** 3-13.

Appendix

Lemmas 2, 3, and 4 are useful in the proofs of Lemma 1 and Theorem 1. Let

$$f_1(e,T) = \overline{v}u(1-e)d'(T)s(e,T)Q^2 - 1$$

$$f_2(\theta,e,T) = [\overline{v}ud(T) - \theta c'(e)]Q - \overline{v}ud(T)[1+(N-1)s(e,T)]Q^2 - \theta F'(e);$$

$f_1(e,T)$ is the first derivative of the manufacturer n's profit function with respect to t_{nm} for $m \in \mathcal{N}\backslash n$ and $f_2(\theta,e,T)$ is the first derivative with respect to e_n, when all manufacturers, including n, choose compliance e and testing per competitor of $t = T/(N-1)$. Define for $T > 0$,

$$\underline{\mathbf{e}}(T) = \left(2d(T) - 1 - \sqrt{1 - 4d(T)/[\overline{v}ud'(T)Q^2]}\right) \Big/ 2d(T)$$

$$\overline{\mathbf{e}}(T) = \left(2d(T) - 1 + \sqrt{1 - 4d(T)/[\overline{v}ud'(T)Q^2]}\right) \Big/ 2d(T),$$

and let $\underline{\mathbf{e}}(0) = \lim_{T\downarrow 0} \underline{\mathbf{e}}(T)$ and $\overline{\mathbf{e}}(0) = \lim_{T\downarrow 0} \overline{\mathbf{e}}(T)$. Let $\overline{T}$ denote the unique solution to

$$\frac{d(T)}{d'(T)} = \frac{\overline{v}uQ^2}{4}.$$

If $T > \overline{T}$, then no value of e satisfies $f_1(e,T) = 0$; otherwise, $f_1(e,T) = 0$ has two roots in e : $\underline{\mathbf{e}}(T)$ and $\overline{\mathbf{e}}(T)$. Note that the notation for the root $\overline{\mathbf{e}}(T)$ is distinct from the upper limit on the manufacturer's compliance investment $\overline{e}$. Note that $f_2(\theta,e,T)$ is strictly increasing in T and strictly decreasing in e. Let

$$\underline{T}(\theta) = \inf_{T\geq 0}\{T : f_2(\theta,0,T) > 0\}.$$

If $T > \underline{T}(\theta)$, then let $\tilde{e}(\theta, T)$ denote the unique solution to

$$f_2(\theta, e, T) = 0,$$

and note that

$$\tilde{e}(\theta, T) > 0; \tag{14}$$

otherwise, let $\tilde{e}(\theta, T) = 0$.

Lemma 2 *Suppose the firms are symmetric and consider the case with RoHS and competitive testing ($t_R = 0$). If*

$$\lim_{T \downarrow 0} \{\overline{v} u d'(T)^3 Q^2 - d'(T)^2 + d(T) d''(T)\} > 0, \tag{15}$$

is violated, then the unique symmetric equilibrium in compliance and testing $(\widehat{e}(\theta, 0), \widehat{T}(\theta, 0)) = (0, 0)$. Otherwise, any symmetric equilibrium $(\widehat{e}(\theta, 0), \widehat{T}(\theta, 0))$ has $\widehat{T}(\theta, 0) \in (0, \overline{T}]$ and one of the following:

$$\widehat{e}(\theta, 0) = \underline{\mathbf{e}}(\widehat{T}) = \tilde{e}(\theta, \widehat{T}) \tag{16}$$

$$\widehat{e}(\theta, 0) = \bar{\mathbf{e}}(\widehat{T}) = \tilde{e}(\theta, \widehat{T}). \tag{17}$$

Moreover, any solution to (16) *or* (17) *is a symmetric equilibrium. Further, $\underline{\mathbf{e}}(\cdot)$ is strictly increasing and $\bar{\mathbf{e}}(\cdot)$ is strictly decreasing; $\tilde{e}(\theta, T)$ is continuous on $T \in [0, \infty)$ and increasing in T, strictly so on $T \in (\underline{T}(\theta), \infty)$. Finally, $\tilde{e}(\theta, 0) = 0$; $\underline{\mathbf{e}}(0) < 0$; and $\bar{\mathbf{e}}(0) > 0$ if and only if* (15).

Proof of Lemma 2: The proof proceeds in five steps. First, we establish necessary conditions for a symmetric equilibrium. Second, we show that these conditions are sufficient. Third, we establish properties of the functions $\tilde{e}(\theta, T)$, $\underline{\mathbf{e}}(T)$ and $\bar{\mathbf{e}}(T)$. Fourth, we show that if (15) is violated, then the unique symmetric equilibrium has zero compliance and testing. Fifth, we show that if (15) is satisfied, then a symmetric equilibrium must satisfy (16) or (17).

First, we establish necessary conditions for a symmetric equilibrium. Recall that $T = (N-1)t$. If manufacturer n anticipates that the remaining manufacturers $m \in \mathcal{N} \backslash n$ will choose compliance $e_m = e$ and testing $t_{mj} = t$ for $j \in \mathcal{N} \backslash m$, then for compliance and testing $(e_n, t_{nm}) = (e, t)$ for $m \in \mathcal{N} \backslash n$ to be a best response for manufacturer n, the following first order conditions must be satisfied

$$(\partial/\partial t_{nm})\pi_n|_{e_i=e,t_{ij}=t \text{ for } i\in\mathcal{N} \text{ and } j\in\mathcal{N}\backslash i} = f_1(e,T) \leq 0 \tag{18}$$

$$(\partial/\partial e_n)\pi_n|_{e_i=e,t_{ij}=t \text{ for } i\in\mathcal{N} \text{ and } j\in\mathcal{N}\backslash i} = f_2(\theta,e,T) \leq 0, \tag{19}$$

where (18) must hold with equality if $T > 0$ and (19) must hold with equality if $e > 0$. That is, a symmetric equilibrium satisfies (18)-(19).

Second, we establish that any solution to (18)-(19) is a symmetric equilibrium. If manufacturer n anticipates that the remaining manufacturers $m \in N\backslash n$ will choose compliance $e_m = e$ and testing $t_{mj} = t$ for $j \in N\backslash m$, then any solution to the first order conditions for manufacturer n must have $t_{nm} = t_n$ for $m \in N\backslash n$. Thus, we can write manufacturer n's expected profit under compliance e_n and total testing expenditure $T_n = (N-1)t_n$ as

$$\pi_n = \overline{v}u\left[1-(N-1)s\left(e,[(N-2)T+T_n]/(N-1)\right)Q-Q\right]s(e_n,T)Q - \theta[c(e_n)Q+F(e_n)] - T_n.$$

Inequalities (5) and (6) together imply

$$d''(t) < -[\overline{v}u(N-1)d'(t)^2]/\{(1-\overline{d})\theta[c''(e)Q+F''(e)]\} \text{ for } (e,\theta)\in[0,\overline{e})\times[\underline{\theta},\overline{\theta}].$$

This, together with the fact that $c(\cdot)$ and $F(\cdot)$ are strictly convex, implies that for any $\theta \in [\underline{\theta},\overline{\theta}]$, π_n is jointly strictly concave in (e_n, T_n), so the first order conditions (18)-(19) are sufficient.

Third, we establish properties of the functions $\tilde{e}(\theta,T)$, $\underline{\mathbf{e}}(T)$ and $\overline{\mathbf{e}}(T)$. Because $d''(T) < -d'(T)^2/d(T)$, $\underline{\mathbf{e}}(\cdot)$ is strictly increasing and $\overline{\mathbf{e}}(\cdot)$ is strictly decreasing. Because $f_2(\theta,\cdot,\cdot)$ is continuous, $\tilde{e}(\theta,T)$ is continuous on $T\in[0,\infty)$. By the implicit function theorem, $\tilde{e}(\theta,T)$ is strictly increasing in T for $T \in (\underline{T}(\theta),\infty)$

$$\frac{\partial\tilde{e}(\theta,T)}{\partial T} = \frac{(\partial/\partial T)f_2(\theta,e,T)}{-(\partial/\partial e)f_2(\theta,e,T)} > 0.$$

By L'Hospital's rule

$$\lim_{T\downarrow 0}\overline{\mathbf{e}}(T) = \lim_{T\downarrow 0}\left(1-\frac{d'(T)^2-d(T)d''(T)}{\overline{v}ud'(T)^3Q^2}\right),$$

so $\overline{\mathbf{e}}(0) > 0$ if and only if (15).

Fourth, suppose (15) is violated. Then $f_1(0,0)\leq 0$ and $f_2(\theta,0,0)\leq 0$, so $(\widehat{e}(\theta,0),\widehat{T}(\theta,0)) = (0,0)$ is an equilibrium; it remains to show that it is unique. Because $\overline{\mathbf{e}}(0)\leq 0$ and $\overline{\mathbf{e}}(\cdot)$ is strictly decreasing, for any $T>0$, $\overline{\mathbf{e}}(T) < 0$. Because $f_1(\cdot,T)$ is strictly concave and $f_1(e,T)=0$ has two roots in e, $\underline{\mathbf{e}}(T)$ and $\overline{\mathbf{e}}(T)$, $f_1(e,T)<0$ for $e>\overline{\mathbf{e}}(T)$. Therefore, for any $e\geq 0$ and $T>0$,

$f_1(e,T) < 0$, which implies that no equilibrium exists with $T > 0$. Because $f_2(\theta,\cdot,0)$ is strictly decreasing, $f_2(\theta,e,0) < 0$ for $e > 0$; this implies that any equilibrium with $T = 0$ must have $e = 0$.

Fifth, suppose (15) holds. Because $f_1(0,0) > 0$, $(e,T) = (0,0)$ is not an equilibrium. Because $f_2(\theta,e,0) < 0$ for $e \in (0,\overline{e})$, an equilibrium cannot have $T = 0$. Thus, in any equilibrium (18) must hold with equality. Thus, a symmetric equilibrium must satisfy $\widehat{T} \in (0,\overline{T}]$, and (16) or (17).■

Lemma 3 *Suppose the firms are symmetric. Under RoHS and competitive testing ($t_R = 0$), for any realized cost multiplier θ, there exists a unique symmetric equilibrium in compliance and testing.*

Proof of Lemma 3: If (15) is violated, then a symmetric equilibrium exists and is unique (from Lemma 2). Therefore, we restrict attention to the case in which (15) holds, which implies $\bar{\mathbf{e}}(0) > 0$ (from Lemma 2). Further, throughout, we restrict attention to $T \in [0,\overline{T}]$, because no symmetric equilibrium exists with $T > \overline{T}$ (from Lemma 2). We next consider three cases and show that in each, there exists a unique symmetric equilibrium.

Case 1: $\underline{T}(\theta) \geq \overline{T}$

First, suppose $\bar{\mathbf{e}}(\overline{T}) = 0$. Then for $T \in [0,\overline{T})$,

$$\bar{\mathbf{e}}(T) > 0 = \tilde{e}(\theta,T) > \underline{\mathbf{e}}(T),$$

so no equilibrium exists with $T \in [0,\overline{T})$ (from Lemma 2). Further,

$$\bar{\mathbf{e}}(\overline{T}) = \underline{\mathbf{e}}(\overline{T}) = \tilde{e}(\theta,\overline{T}) = 0,$$

so the unique equilibrium is $(\widehat{e}(\theta,0),\widehat{T}(\theta,0)) = (0,\overline{T})$ (from Lemma 2).

Second, suppose $\bar{\mathbf{e}}(\overline{T}) < 0$. Then for $T \in [0,\overline{T}]$,

$$\tilde{e}(\theta,T) \geq 0 > \bar{\mathbf{e}}(\overline{T}) = \underline{\mathbf{e}}(\overline{T}) \geq \underline{\mathbf{e}}(T),$$

so there does not exist a solution to (16). However, because $\tilde{e}(\theta,T) = 0$ for $T \in [0,\overline{T}]$, $\bar{\mathbf{e}}(0) > 0$, $\bar{\mathbf{e}}(\overline{T}) < 0$, and $\bar{\mathbf{e}}(\cdot)$ is strictly decreasing, there exists a unique solution to (17) and this is the unique equilibrium (by Lemma 2).

Third, suppose $\bar{\mathbf{e}}(\overline{T}) > 0$. By similar argument, there does not exist a solution to (17), but there exists a unique solution to (16) and this is the unique equilibrium.

Case 2: $\underline{T}(\theta) < \overline{T}$ and $\bar{\mathbf{e}}(\underline{T}(\theta)) \leq 0$

Because for $T \in [0,\overline{T}]$,

$$\tilde{e}(\theta,T) \geq 0 \geq \bar{\mathbf{e}}(\underline{T}(\theta)) > \bar{\mathbf{e}}(\overline{T}) = \underline{\mathbf{e}}(\overline{T}) \geq \underline{\mathbf{e}}(T),$$

there does not exist a solution to (16). However, because $\bar{\mathbf{e}}(0) > 0 = \tilde{e}(\theta, 0)$, $\bar{\mathbf{e}}(\underline{T}(\theta)) \leq 0 = \tilde{e}(\theta, \underline{T}(\theta))$, $\bar{\mathbf{e}}(\cdot)$ is strictly decreasing, and $\tilde{e}(\theta, \cdot)$ is increasing, there exists a unique solution to (17) and this is the unique equilibrium (by Lemma 2).

CASE 3: $\underline{T}(\theta) < \overline{T}$ AND $\bar{\mathbf{e}}(\underline{T}(\theta)) > 0$

The proof for this case proceeds in four steps. First, we show that no symmetric equilibrium exists with $T \leq \underline{T}(\theta)$, which allows us in subsequent steps to restrict attention to $T \in (\underline{T}(\theta), \overline{T}]$. Second, we show that if $f_2(\theta, \underline{\mathbf{e}}(T), T) = 0$ has no roots, then the unique symmetric equilibrium has (17). Third, we show if $f_2(\theta, \underline{\mathbf{e}}(T), T) = 0$ has one root, then the unique symmetric equilibrium has (16). Fourth, we show that $f_2(\theta, \underline{\mathbf{e}}(T), T) = 0$ has at most one root.

First, we establish that

$$d(\underline{T}(\theta)) < 1/2. \tag{20}$$

Inequality (20) is immediate if $\overline{d} \leq 1/2$; otherwise (20) follows from the fact that $f_2(\theta, 0, T)$ is strictly increasing in T and

$$f_2(\theta, 0, T)|_{d(T)=1/2} > \overline{v}u(N-1)Q^2/4 > 0, \tag{21}$$

where the first inequality follows from (7). Inequality (20) implies

$$\underline{\mathbf{e}}(\underline{T}(\theta)) < 0. \tag{22}$$

Because $\tilde{e}(\theta, \underline{T}(\theta)) = 0$, (22) implies that $\underline{\mathbf{e}}(\underline{T}(\theta)) < \tilde{e}(\theta, \underline{T}(\theta))$, or equivalently,

$$f_2(\theta, \underline{\mathbf{e}}(\underline{T}(\theta)), 0) > 0. \tag{23}$$

Because for $T \leq \underline{T}(\theta)$, $\underline{\mathbf{e}}(T) < \tilde{e}(\theta, \underline{T}(\theta)) = 0 < \bar{\mathbf{e}}(\underline{T}(\theta)) \leq \bar{\mathbf{e}}(T)$, there does not exist a solution to either (16) or (17), and therefore no symmetric equilibrium with $T \leq \underline{T}(\theta)$ exists (from Lemma 2). Because no symmetric equilibrium with $T \leq \underline{T}(\theta)$ exists, in the subsequent steps we restrict attention to $T \in (\underline{T}(\theta), \overline{T}]$.

Second, suppose that $f_2(\theta, \underline{\mathbf{e}}(T), T) = 0$ has no roots. This, together with (23) and the observation that $\underline{\mathbf{e}}(\cdot)$ and $f_2(\theta, \cdot, \cdot)$ are continuous implies $f_2(\theta, \underline{\mathbf{e}}(T), T) > 0$, or equivalently,

$$\underline{\mathbf{e}}(T) < \tilde{e}(\theta, T). \tag{24}$$

Because $\underline{\mathbf{e}}(\overline{T}) = \bar{\mathbf{e}}(\overline{T})$, this implies

$$\bar{\mathbf{e}}(\overline{T}) < \tilde{e}(\theta, \overline{T}). \tag{25}$$

Because $\tilde{e}(\theta, \cdot)$ is strictly increasing, $\bar{\mathbf{e}}(\cdot)$ is strictly decreasing, and $\bar{\mathbf{e}}(\underline{T}(\theta)) > \tilde{e}(\theta, \underline{T}(\theta))$, (25) implies that there is a unique $T \in (\underline{T}(\theta), \overline{T})$ such that

$$\bar{\mathbf{e}}(T) = \tilde{e}(\theta, T).$$

This observation, together with (24) and Lemma 2, implies that the unique symmetric equilibrium has (17).

Third, suppose that $f_2(\theta, \underline{\mathbf{e}}(T), T) = 0$ has one root. Then (23) implies $f_2(\theta, \underline{\mathbf{e}}(\overline{T}), \overline{T}) \leq 0$, or equivalently,

$$\underline{\mathbf{e}}(\overline{T}) \geq \tilde{e}(\theta, \overline{T}).$$

Because $\bar{\mathbf{e}}(T) > \underline{\mathbf{e}}(\overline{T})$ for $T < \overline{T}$, this implies

$$\bar{\mathbf{e}}(T) > \tilde{e}(\theta, \overline{T})$$

for $T < \overline{T}$. This observation, together with Lemma 2 and the observation that $\underline{\mathbf{e}}(T) = \tilde{e}(\theta, T)$ has a unique solution, implies that the unique symmetric equilibrium has (16).

Fourth, we show that there exists $\underline{d} > 0$ such that $d''(T) < -\underline{d}$ implies that $f_2(\theta, \underline{\mathbf{e}}(T), T) = 0$ has at most one root. Because of (23), it is sufficient to show that $f_2(\theta, \underline{\mathbf{e}}(\cdot), \cdot)$ is strictly concave. Straightforward, albeit tedious algebra establishes that $c'''(e)Q + F'''(e) \geq 0$ implies

$$\lim_{d \to -\infty} \frac{\partial^2}{\partial T^2} f_2(\theta, \underline{\mathbf{e}}(T), T)|_{d''(T)=d} < 0.$$

Because $\frac{\partial^2}{\partial T^2} f_2(\theta, \underline{\mathbf{e}}(T), T)|_{d''(T)=d}$ is continuous in d, there exists $\underline{d} > 0$ such that $d''(T) < -\underline{d}$ implies that $f_2(\theta, \underline{\mathbf{e}}(T), T)$ is strictly concave.■

Lemma 4 *Suppose the firms are symmetric and that in the case with RoHS and competitive testing* ($t_R = 0$), *the symmetric equilibrium is unique. Then, if the regulator tests* $t_R > 0$, *the unique symmetric equilibrium is*

$$\widehat{T}(\theta, t_R) = \left[\widehat{T}(\theta, 0) - \alpha t_R\right]^+ \tag{26}$$

$$\widehat{e}(\theta, t_R) = \tilde{e}(\theta, \max(\widehat{T}(\theta, 0), \alpha t_R)).$$

Proof of Lemma 4: If each manufacturer applies t^f to its $N-1$ competitors, then the total testing expenditure applied to each firm is $T^f = (N-1)t^f$. If the manufacturers collectively apply testing expenditure T^f to each manufacturer and the regulator applies testing expenditure t_R to each manufacturer, the compliance effort in a symmetric equilibrium is $\tilde{e}(\theta, \alpha t_R + T^f)$.

Because in the base case with no regulator testing there is a unique symmetric equilibrium with compliance and firm testing $(\widehat{e}(\theta,0),\widehat{T}(\theta,0))$, if the regulator announces testing $t_R \le \widehat{T}(\theta,0)/\alpha$, then in the unique symmetric equilibrium the firms apply testing expenditure $T^f = \widehat{T}(\theta,0) - \alpha t_R$ and choose compliance $\widehat{e}(\theta,0)$.

If the regulator announces testing $t_R > \widehat{T}(\theta,0)/\alpha$, then a symmetric equilibrium will have compliance weakly larger than $\widehat{e}(\theta,0)$ (this follows from $\alpha t_R + T^f > \widehat{T}(\theta,0)$ and $\tilde{e}(\theta,T)$ being increasing in T). Suppose that the firms apply strictly positive testing expenditure $T^f > 0$; this implies that there exists an equilibrium in the base case with no regulator testing with compliance weakly greater than $\widehat{e}(\theta,0)$ and testing strictly greater than $\widehat{T}(\theta,0)$, which contradicts that the symmetric equilibrium in the base case is unique. We conclude that in equilibrium, if the regulator announces testing $t_R > \widehat{T}(\theta,0))/\alpha$, the firms do not test $T^f = 0$.■

Proof of Lemma 1: All but (8) is immediate from Lemmas 3 and 4. Inequality (8) follows from Lemma 4 and that observation that $\tilde{e}(\theta,T)$ is increasing in T (from Lemma 2).■

Proof of Theorem 1: The proof proceeds in six steps. In Step 1, we establish some properties of the expected social welfare function. In Steps 2 to 5, we compare the regulator's preference over the three alternatives: imposing RoHS with regulator testing, imposing RoHS without regulator testing, and not imposing RoHS. In Step 6, we establish sufficient conditions for $\underline{x} < \overline{x}$

Step 1: Properties of the regulator's objective function

Consider the relaxed problem in which the regulator first dictates the level of testing, and the individual manufacturers then simultaneously choose compliance. If the regulator chooses testing level T per firm, the manufacturers in the unique symmetric equilibrium choose compliance $\tilde{e}(\theta,T)$. The expected social welfare that results is

$$P(\theta,x,T) \equiv E[\overline{v}u\Sigma_{n=1}^N \widetilde{Q}_n(1-\Sigma_{n=1}^N \widetilde{Q}_n/2)] - N(\theta[c(\tilde{e}(\theta,T))Q + F(\tilde{e}(\theta,T))] - xN[1-\tilde{e}(\theta,T)]Q - NT,$$

where $\tilde{e}(\theta,T)$ and T determine the distribution of $\widetilde{Q}_n : \widetilde{Q}_n = Q$ with probability $s(\tilde{e}(\theta,T),T)$ and $\widetilde{Q}_n = 0$ otherwise.

Recall that expected social welfare under RoHS, where the regulator first chooses testing level t_R and the manufacturers follow by choosing compliance and testing levels, is given by (9), which we denote $S(x,t_R)$. From Lemma 4 and the observation that $\widehat{e}(\theta,0) = \tilde{e}(\theta,\widehat{T}(\theta,0))$,

$$S(x,t_R) = E_\theta[P(\theta,x,\max(\widehat{T}(\theta,0),\alpha t_R)) - N(1-\alpha)t_R]. \tag{27}$$

From Lemma 2, for every $\theta \in [\underline{\theta},\overline{\theta}]$, $\tilde{e}(\theta,T)$ is increasing in T, strictly so for $T > \underline{T}(\theta)$, so the term $-N(1-\tilde{e}(\theta,T))Q$ in $P(\theta,x,T)$ is increasing in T, strictly so for $T > \underline{T}(\theta)$. Intuitively, an increase in testing reduces the expected quantity of noncompliant production. Furthermore, as the environmental cost parameter x increases, the increase in $-xN(1-\tilde{e}(\theta,T))Q$ with respect to testing T strictly increases. Therefore, expected social welfare $P(\theta,x,T)$ is supermodular in x and T, and is strictly so for $T > \underline{T}(\theta)$. It immediately follows that $S(x,t_R)$ is supermodular in x and t_R.

Step 2: Regulator's preference for imposing RoHS with testing vs. imposing RoHS without testing

In this step, we compare the regulator's preference for imposing RoHS and testing versus imposing RoHS and not testing. Let

$$x_3 = \sup\{x : S(x,0) \geq S(x,t_R) \text{ for } t_R > 0\}.$$

Then for any $x > x_3$, there exists $t_R > 0$ such that

$$S(x,t_R) > S(x,0),$$

so the regulator prefers to impose RoHS and test rather than impose RoHS and not test for $x > x_3$. Because $\tilde{e}(\theta,\cdot)$ is continuous, $S(\cdot,\cdot)$ is continuous. This implies

$$S(x_3,0) \geq S(x_3,t_R) \text{ for } t_R > 0,$$

so, by convention, the regulator prefers to impose RoHS and not test rather than impose RoHS and test for $x = x_3$. For any $x < x_3$ and any $t_R > 0$,

$$\begin{aligned} 0 &\geq S(x_3,t_R) - S(x_3,0) \\ &\geq S(x,t_R) - S(x,0), \end{aligned}$$

where the second inequality follows from supermodularity of $S(\cdot,\cdot)$. Therefore, by convention, the regulator prefers to impose RoHS and not test rather than impose RoHS and test for $x < x_3$.

If $\Pr(\widehat{T}(\theta,0) > 0) = 0$, then social welfare is identical when the regulator does not impose RoHS and when the regulator imposes RoHS without testing, so by convention the regulator does not impose RoHS without testing. Therefore the statement of the proposition regarding when the manufacturer imposes RoHS and when the manufacturer tests holds with $\underline{x} = \overline{x} = x_3$. In the sequel, we assume that $\Pr(\widehat{T}(\theta,0) > 0) > 0$.

Step 3: Regulator's preference for imposing RoHS with testing vs. not imposing

RoHS

In this step, we compare the regulator's preference for imposing RoHS with testing versus not imposing RoHS. Let

$$x_2 = \sup\{x : E_\theta[P(\theta, x, 0)] \geq S(x, t_R) \text{ for } t_R > 0\}.$$

Then for any $x > x_2$, there exists $t_R > 0$ such that

$$S(x, t_R) > E_\theta[P(\theta, x, 0)],$$

so the regulator prefers to impose RoHS and test rather than not impose RoHS for $x > x_2$.

Because $S(\cdot, \cdot)$ and $E_\theta[P(\theta, \cdot, 0)]$ are continuous,

$$E_\theta[P(\theta, x_2, 0)] \geq S(x_2, t_R) \text{ for } t_R > 0,$$

so, by convention, the regulator prefers to not impose RoHS rather than impose RoHS and test for $x = x_2$. For any $x < x_2$ and any $t_R > 0$,

$$\begin{aligned}
0 &\geq S(x_2, t_R) - E_\theta[P(\theta, x_2, 0)] \\
&= S(x_2, t_R) - S(x_2, 0) + E_\theta[P(\theta, x_2, \widehat{T}(\theta, 0))] - E_\theta[P(\theta, x_2, 0)] \\
&\geq S(x, t_R) - S(x, 0) + E_\theta[P(\theta, x, \widehat{T}(\theta, 0))] - E_\theta[P(\theta, x, 0)] \\
&= S(x, t_R) - E_\theta[P(\theta, x, 0)],
\end{aligned}$$

where the equalities follow from (27) and the second inequality follows from supermodularity of $S(\cdot, \cdot)$ and $P(\theta, \cdot, \cdot)$. Therefore, by convention, the regulator prefers to not impose RoHS rather than impose RoHS and test for $x < x_2$.

Step 4: Regulator's preference for imposing RoHS without testing vs. not imposing RoHS

In this step, we compare the regulator's preference for imposing RoHS without testing versus not imposing RoHS. First, suppose that $\Pr(\widehat{e}(\theta, 0) > 0) > 0$, or equivalently $\Pr(\widehat{T}(\theta, 0) > \underline{T}(\theta)) > 0$. In this case, let x_1 denote the unique solution to

$$E_\theta[P(\theta, x, \widehat{T}(\theta, 0))] = E_\theta[P(\theta, x, 0)].$$

By convention, the regulator prefers to not impose RoHS rather than impose RoHS without testing for $x = x_1$. For $x > x_1$,

$$E_\theta[P(\theta, x, \widehat{T}(\theta, 0))] - E_\theta[P(\theta, x, 0)]$$

$$> E_\theta[P(\theta, x_1, \widehat{T}(\theta, 0))] - E_\theta[P(\theta, x_1, 0)] = 0,$$

where the inequality follows because $P(\theta, \cdot, \cdot)$ is strictly supermodular for θ such that $\widehat{T}(\theta, 0) > \underline{T}(\theta)$. Thus, the regulator prefers to impose RoHS without testing rather than to not impose RoHS for $x > x_1$. For $x < x_1$,

$$0 = E_\theta[P(\theta, x_1, \widehat{T}(\theta, 0))] - E_\theta[P(\theta, x_1, 0)]$$

$$> E_\theta[P(\theta, x, \widehat{T}(\theta, 0))] - E_\theta[P(\theta, x, 0)],$$

so the regulator prefers not to impose RoHS rather than to impose RoHS without testing for $x < x_1$.

Second, suppose that $\Pr(\widehat{e}(\theta, 0) > 0) = 0$. Then imposing RoHS without regulator testing only has the effect of causing the system to incur testing costs and of reducing the expected utility generated by sold units, without the benefit of increased compliance. Therefore, the regulator always prefers not to impose RoHS than rather to impose RoHS without testing. In this case, we define $x_1 = \infty$, so that we can say, consistent with the first case, that the regulator prefers not to impose RoHS rather than to impose RoHS with testing if and only if $x < x_1$.

Step 5: Regulator's overall preference

If $x_1 \leq x_3$, then the the statement of the proposition regarding when the manufacturer imposes RoHS and when the manufacturer tests holds with $\underline{x} = x_1$ and $\overline{x} = x_3$. If $x_1 > x_3$, then $x_2 \in [x_3, x_1]$ and the statement holds with $\underline{x} = \overline{x} = x_2$.

Step 6: Sufficient conditions for $\underline{x} < \overline{x}$

Note that $\Pr(\widehat{e}(\theta, 0) > 0) > 0$ implies $\Pr(\widehat{T}(\theta, 0) > 0) > 0$ and $x_1 < \infty$ (from Step 4). Further, observe that x_1 is invariant to α. Because as $\alpha \to 0$, $x_3 \to \infty$, there exists $\overline{\alpha} \in (0, 1]$ such that if $\alpha < \overline{\alpha}$, $x_1 < x_3$, which (by Step 5) implies $\underline{x} < \overline{x}$. The condition $\alpha < \overline{\alpha}$ is not necessary for $\underline{x} < \overline{x}$. Examples with $\alpha = 1$ and $\underline{x} < \overline{x}$ appear in Figure 1.■

Proof of Proposition 1: First, we demonstrate the comparative statics for the number of firms N. Because we have normalized $\theta = 1$, we suppress the dependence of $\tilde{e}$, $\widehat{e}$ and f_2 on θ. By the implicit function theorem, $\tilde{e}(T)$ is decreasing in N. Let

$$\tilde{N} = \max_{N \in \{2,3,..\}} \left\{ N : \tilde{e}(\overline{T}) \geq 1 - \frac{1}{2d(\overline{T})} \right\}.$$

With some abuse of notation, let $(\widehat{e}(N), \widehat{T}(N))$ denote the unique symmetric equilibrium. If $N \leq \tilde{N}$, then $\widehat{T}$ is the unique solution to

$$\bar{\mathbf{e}}(T) - \tilde{e}(T) = 0.$$

Further,

$$\bar{\mathbf{e}}(T) - \tilde{e}(T) \geq 0 \text{ if and only if } T \in [0, \widehat{T}]. \tag{28}$$

For any $N_0 < N_1 \leq \tilde{N}$,

$$\begin{aligned} 0 &= \left[\bar{\mathbf{e}}(\widehat{T}(N_0)) - \tilde{e}(\widehat{T}(N_0))\right] |_{N=N_0} \\ &\leq \left[\bar{\mathbf{e}}(\widehat{T}(N_0)) - \tilde{e}(\widehat{T}(N_0))\right] |_{N=N_1}, \end{aligned}$$

where the inequality follows because $\tilde{e}(T)$ is decreasing in N. This implies that

$$\widehat{T}(N_0) \leq \widehat{T}(N_1) \tag{29}$$

(from (28)). Thus,

$$\widehat{e}(N_0) = \bar{\mathbf{e}}(\widehat{T}(N_0)) \geq \bar{\mathbf{e}}(\widehat{T}(N_1)) = \widehat{e}(N_1), \tag{30}$$

where the inequality follows from (29) and the fact that $\bar{\mathbf{e}}(\cdot)$ is decreasing. By similar argument, for any $\tilde{N} < N_2 < N_3$,

$$\widehat{T}(N_2) \geq \widehat{T}(N_3) \tag{31}$$

$$\widehat{e}(N_2) \geq \widehat{e}(N_3). \tag{32}$$

Further, for any $N_1 \leq \tilde{N} < N_2$,

$$\widehat{e}(N_1) \geq 1 - \frac{1}{2d(\overline{T})} \geq \widehat{e}(N_2). \tag{33}$$

Together (30), (32) and (33) imply that $\widehat{e}$ is decreasing in N. Together (29) and (31) imply that $\widehat{T}$ is increasing in N for $N \leq \overline{N}$ and decreasing in N for $N \geq \overline{N}$, where $\overline{N} = \arg\max_{N \in \{2,3,..\}}\{\widehat{T}\}$.

Second, we demonstrate the comparative statics for the quality level u. Note that $\underline{\mathbf{e}}(T)$ is strictly decreasing in u, $\bar{\mathbf{e}}(T)$ is increasing in u, and by the implicit function theorem, $\tilde{e}(T)$ is increasing in u. If $1 - 1/[2d(\overline{T})] > 0$, then let $\tilde{u}$ denote the unique value of u such that

$$\tilde{e}(\overline{T}) = 1 - \frac{1}{2d(\overline{T})},$$

and otherwise, let $\tilde{u} = 0$. With some abuse of notation, let $(\widehat{e}(u), \widehat{T}(u))$ denote the unique symmetric equilibrium. For any $\{u_a, u_b\} < \tilde{u}$, let $\tilde{T}(u_a, u_b)$ denote a solution to

$$\underline{\mathbf{e}}(T)|_{u=u_a} - \tilde{e}(T)|_{u=u_b} = 0. \tag{34}$$

Note that when $u_a = u_b = u$, there is only one solution to (34) and $\widehat{T}(u) = \tilde{T}(u, u)$. For any $u_0 < u_1 < \tilde{u}$, we will establish that

$$\widehat{T}(u_0) \leq \tilde{T}(u_0, u_1). \tag{35}$$

The proof is by contradiction. Suppose that $\widehat{T}(u_0) > \tilde{T}(u_0, u_1)$. Then

$$\begin{aligned} \underline{\mathbf{e}}(\tilde{T}(u_0, u_1))|_{u=u_0} &< \tilde{e}(\tilde{T}(u_0, u_1))|_{u=u_0} \\ &\leq \tilde{e}(\tilde{T}(u_0, u_1))|_{u=u_1}, \end{aligned} \tag{36}$$

where the first inequality holds because of the continuity of $\underline{\mathbf{e}}(\cdot)$ and $\tilde{e}(\cdot)$, $\underline{\mathbf{e}}(0) < \tilde{e}(0)$, and uniqueness of $(\widehat{e}, \widehat{T})$ imply $[\underline{\mathbf{e}}(T) - \tilde{e}(T)]|_{u=u_0} < 0$ for $T \in [0, \widehat{T})$; the second inequality holds because $\tilde{e}(T)$ is increasing in u. Because (36) contradicts the definition of $\tilde{T}(u_0, u_1)$, we have established (35). By similar argument,

$$\tilde{T}(u_0, u_1) \leq \widehat{T}(u_1). \tag{37}$$

We conclude that

$$\widehat{e}(u_0) = \underline{\mathbf{e}}(\widehat{T}(u_0))|_{u=u_o} \leq \underline{\mathbf{e}}(\tilde{T}(u_0, u_1))|_{u=u_o} = \tilde{e}(\tilde{T}(u_0, u_1))|_{u=u_1} \leq \tilde{e}(\widehat{T}(u_1))|_{u=u_1} = \widehat{e}(u_1), \tag{38}$$

where the first inequality follows from (35) and $\underline{\mathbf{e}}(\cdot)$ being increasing; the second inequality follows from (37) and $\tilde{e}(\cdot)$ being increasing. By similar argument, for any $\tilde{u} < u_2 < u_3$,

$$\widehat{e}(u_2) \leq \widehat{e}(u_3). \tag{39}$$

Because $\widehat{e}(u)$ is continuous in u, (38) and (39) imply that $\widehat{e}$ is increasing in u.■

Internet Appendix

Proof of Proposition 2: We adopt the abbreviated notation for success probability $s_n \equiv s(e_n, \Sigma_{j\in\mathcal{N}\backslash n} t_{jn})$ and note that

$$s_n \geq 1 - \overline{d} > 0 \text{ for } n \in \mathcal{N}. \tag{40}$$

Any Nash equilibrium must satisfy the following first order necessary conditions for the optimality of manufacturer n's compliance and testing strategy. For every $n \in \{1, .., N\}$ and $m \in \{1, .., N\}\backslash n$,

$$\frac{\partial \pi_n}{\partial e_n} = \overline{v}[u_n(1 - \Sigma_{m=n+1}^{N} s_m Q_m - Q_n) - \Sigma_{m=1}^{n-1} u_m s_m Q_m] d\left(\Sigma_{m\in\mathcal{N}\backslash n} t_{mn}\right) Q_n - c'(e_n)Q_n - F'(e_n) \leq 0 \tag{41}$$

$$\frac{\partial \pi_n}{\partial t_{nm}} = \overline{v} \min(u_n, u_m) d'(\Sigma_{j\in\mathcal{N}\backslash m} t_{jm})(1 - e_m) Q_m s_n Q_n - 1 \leq 0, \tag{42}$$

where (41) must hold with equality if $e_n > 0$ and (42) must hold with equality if $t_{nm} > 0$.

To establish (11), we can choose

$$u_L \leq \min_{l\in 1,..,N_L} \left[\frac{F'(0) + c'(0)Q_l}{\overline{v}(1 - \Sigma_{m=l+1}^{N}(1 - \overline{d})Q_m - Q_l)} \right]. \tag{43}$$

Then for $l \in \{1, .., N_L\}$, $u_l \leq u_L$, (40) and (41) imply that $e_l = 0$. Our assumptions that $F'(0) + c'(0) > 0$, $Q_n > 0$ and $\Sigma_{n=1}^{N} Q_n < 1$ guarantee that the right hand side of (43) is strictly positive.

Next, we will establish (13). Our assumption that $\lim_{e_n \uparrow \overline{e}} [c(e_n) + F(e_n)] = \infty$ for $\overline{e} \in (0, 1)$ guarantees that

$$e_n < \overline{e} < 1. \tag{44}$$

With (40), (44) and our assumptions $u_n > 0$, $Q_n > 0$ and $\lim_{t\downarrow 0} d'(t) = \infty$, the first order condition on testing (42) implies that

$$\Sigma_{j\in\mathcal{N}\backslash m} t_{jm} > 0 \text{ for every } m \in \mathcal{N}$$

and

$$\max_{n\in\mathcal{N}\backslash m} \left[\overline{v} \min(u_n, u_m) d'(\Sigma_{j\in\mathcal{N}\backslash m} t_{jm})(1 - e_m) Q_m s_n Q_n\right] = 1 \text{ for every } m \in \mathcal{N}. \tag{45}$$

To use (45), the first order condition on testing (42) and concavity of the detection function $d(\cdot)$

to establish (13), we need to show that for any $l \in \{1, .., N_L\}$ and $h \in \{N_L + 1, .., N\}$

$$\max_{n \in \mathcal{N} \setminus l} [\min(u_n, u_l)(1 - e_l) Q_l s_n Q_n] < \max_{n \in \mathcal{N} \setminus h} [\min(u_n, u_h)(1 - e_h) Q_h s_n Q_n] . \tag{46}$$

In words, the maximal manufacturer's incentive for testing manufacturer l is strictly lower than the maximal manufacturer's incentive for testing manufacturer h. Because $e_l = 0$, the left hand side of (46) satisfies

$$\max_{n \in \mathcal{N} \setminus l} [\min(u_n, u_l)(1 - e_l) Q_l s_n Q_n] \leq u_l \max_{n \in \{1, .., N_L\}} [Q_n] \max_{n \in \mathcal{N}} [Q_n]. \tag{47}$$

We can choose

$$u_L < u_{N_L+1}(1 - \overline{e})(1 - \overline{d}) \frac{\min_{n \in \{N_L+1, .., N\}} [Q_n]}{\max_{n \in \{1, .., N_L\}} [Q_n]}. \tag{48}$$

Our assumptions that $u_n > 0$, $Q_n > 0$, $\overline{d} < 1$ and $\overline{e} < 1$ make the right hand side of (48) strictly positive. Then $u_l \leq u_L$ implies

$$\arg\max_{n \in \mathcal{N} \setminus h} [\min(u_n, u_h)(1 - e_h) Q_h s_n Q_n] \in \{N_L + 1, .., N\}. \tag{49}$$

Using (40), (44) and (49), the right hand side of (46) satisfies

$$\max_{n \in \mathcal{N} \setminus h} [\min(u_n, u_h)(1 - e_h) Q_h s_n Q_n] \geq u_{N_L+1}(1 - \overline{e})(1 - \overline{d}) \left(\min_{n \in \{N_L+1, .., N\}} [Q_n] \right)^2 .$$

Therefore (46) holds if

$$u_l \max_{n \in \{1, .., N_L\}} [Q_n] \max_{n \in \mathcal{N}} [Q_n] < u_{N_L+1}(1 - \overline{e})(1 - \overline{d}) \left(\min_{n \in \{N_L+1, .., N\}} [Q_n] \right)^2 .$$

We can choose

$$u_L < u_{N_L+1}(1 - \overline{e})(1 - \overline{d}) \frac{\left(\min_{n \in \{N_L+1, .., N\}} [Q_n] \right)^2}{\max_{n \in \{1, .., N_L\}} [Q_n] \max_{n \in \mathcal{N}} [Q_n]}. \tag{50}$$

Then $u_l \leq u_L$ implies (46) and hence (13). Our assumptions that $u_n > 0$, $Q_n > 0$, $\overline{d} < 1$ and $\overline{e} < 1$ make the right hand side of (50) strictly positive.

Similarly, to use (45) and the first order condition on testing (42) to establish (12), we need to show that for every $l \in \{1, .., N_L\}$ and $h \in \{N_L + 1, .., N\}$,

$$\min(u_l, u_h) s_l Q_l < \max_{n \in \mathcal{N} \setminus h} [\min(u_n, u_h) s_n Q_n] . \tag{51}$$

In words, some manufacturer with index in $\{N_L + 1, .., N\}$ has strictly greater incentive to test manufacturer h than does manufacturer l. Inequality (51) will ensure that the inequality in (42) is strict for $(n, m) = (l, h)$. The left hand side of (51) satisfies

$$\min(u_l, u_h) s_l Q_l \leq u_l \max_{n \in \{1,..,N_L\}} [Q_n].$$

Together (48) and $u_l \leq u_L$ imply

$$\arg\max_{n \in \mathcal{N} \setminus h} [\min(u_n, u_h) s_n Q_n] \in \{N_L + 1, .., N\}, \tag{52}$$

so the right hand side of (51) satisfies

$$\max_{n \in \mathcal{N} \setminus h} [\min(u_n, u_h) s_n Q_n] \geq u_{N_L+1}(1 - \overline{d}) \max_{n \in \{N_L+1,..,N\}} [Q_n].$$

Therefore (51) holds if

$$u_l \max_{n \in \{1,..,N_L\}} [Q_n] < u_{N_L+1}(1 - \overline{d}) \max_{n \in \{N_L+1,..,N\}} [Q_n].$$

We can choose

$$u_L < u_{N_L+1}(1 - \overline{d}) \frac{\max_{n \in \{N_L+1,..,N\}} [Q_n]}{\max_{n \in \{1,..,N_L\}} [Q_n]}. \tag{53}$$

Then $u_l \leq u_L$ implies (51) and hence (12). Our assumptions that $u_n > 0$, $Q_n > 0$, and $\overline{d} < 1$ make the right hand side of (53) strictly positive.

We will conclude the proof by showing that manufacturer l has strictly greater expected profit due to regulation. Observe for each firm $h \in \{N_L + 1, .., N\}$,

$$\max_{n \in \mathcal{N} \setminus h} [\overline{v} \min(u_n, u_h)(1 - e_h) Q_h s_n Q_n] \geq \overline{v} u_{N_L+1}(1 - \overline{e})(1 - \overline{d}) \left(\max_{n \in \{N_L+1,..,N\}} [Q_n] \right)^2,$$

and let $\underline{t}$ denote the unique solution to

$$d'(\underline{t}) = \left[\overline{v} u_{N_L+1}(1 - \overline{e})(1 - \overline{d}) \left(\max_{n \in \{N_L+1,..,N\}} [Q_n] \right)^2 \right]^{-1}. \tag{54}$$

Then (45), the first order condition on testing (42) and strict concavity of the detection function $d(\cdot)$ guarantee that testing of manufacturer h satisfies

$$\Sigma_{j \in \mathcal{N} \setminus h}\, t_{jh} \geq \underline{t} > 0$$

and with (44) that

$$s_h \leq 1 - d(\underline{t})(1 - \overline{e}) \equiv \overline{s} < 1. \tag{55}$$

Recall (47) and let $\bar{t}$ denote the unique solution to

$$d'(\bar{t}) = \left[u_L \max_{n\in\{1,..,N_L\}}[Q_n]\max_{n\in\mathcal{N}}[Q_n]\right]^{-1}.$$

Then (45), the first order condition on testing (42), and strict concavity of the detection function $d(\cdot)$ guarantee that for $l \in \{1,..,N_L\}$ with $u_l \leq u_L$, testing of manufacturer l satisfies

$$\Sigma_{j\in\mathcal{N}\backslash l}\ t_{jl} \leq \bar{t}.$$

Because $d(0) = 0$, we can choose u_L strictly positive but sufficiently small that

$$d\left(\Sigma_{j\in\mathcal{N}\backslash l}\ t_{jl}\right) \leq d(\bar{t}) < \Sigma_{m=N_L+1}^{N}(1-\bar{s})Q_m. \tag{56}$$

With no regulation, manufacturer $l \in \{1,..,N_L\}$ would have profit

$$\pi_l^{NR} = \bar{v}\left[u_l\left(1-\Sigma_{m=l}^{N}Q_m\right) - \Sigma_{m=1}^{l-1}u_mQ_m\right]Q_l - c(0)Q_l.$$

With regulation and $u_l \leq u_L$, manufacturer l sets $e_l = 0$ and $t_{lh} = 0$ for all $h \in \{N_L+1,..,N\}$. Manufacturer l may also choose not to test other firms with low quality, i.e., to set $t_{lm} = 0$ for all $m \in \{1,..,N_L\}\backslash l$. (In constructing a lower bound on manufacturer l's profit under regulation, we will assume $s_m = 1$ for $m \in \{1,..,N_L\}\backslash l$ so the decision to set $t_{lm} = 0$ maximizes that lower bound.) Therefore, regulation increases manufacturer l's expected profit by

$$\begin{aligned}
\pi_l - \pi_l^{NR} &\geq \bar{v}\left[u_l\left(1-\Sigma_{m=N_L+1}^{N}s_mQ_m - \Sigma_{m=l}^{N_L}Q_m\right) - \Sigma_{m=1}^{l-1}u_mQ_m\right]s_lQ_l \\
&\quad - \bar{v}\left[u_l\left(1-\Sigma_{m=l}^{N}Q_m\right) - \Sigma_{m=1}^{l-1}u_mQ_m\right]Q_l \\
&\geq \bar{v}u_lQ_l[(s_l-1) + \Sigma_{m=N_L+1}^{N}Q_m(1-s_ms_l))] \\
&\geq \bar{v}u_lQ_l[\Sigma_{m=N_L+1}^{N}Q_m(1-\bar{s}) - d(\Sigma_{j\in\mathcal{N}\backslash l}\ t_{jl})] \\
&> 0,
\end{aligned}$$

where the third inequality follows from $s_l = 1 - d(\Sigma_{j\in\mathcal{N}\backslash l}\ t_{jl}) < 1$ and (55) and the final strict inequality follows from (56).■