HIERARCHICAL (MULTI-SCALE) MODELING FRAMEWORK

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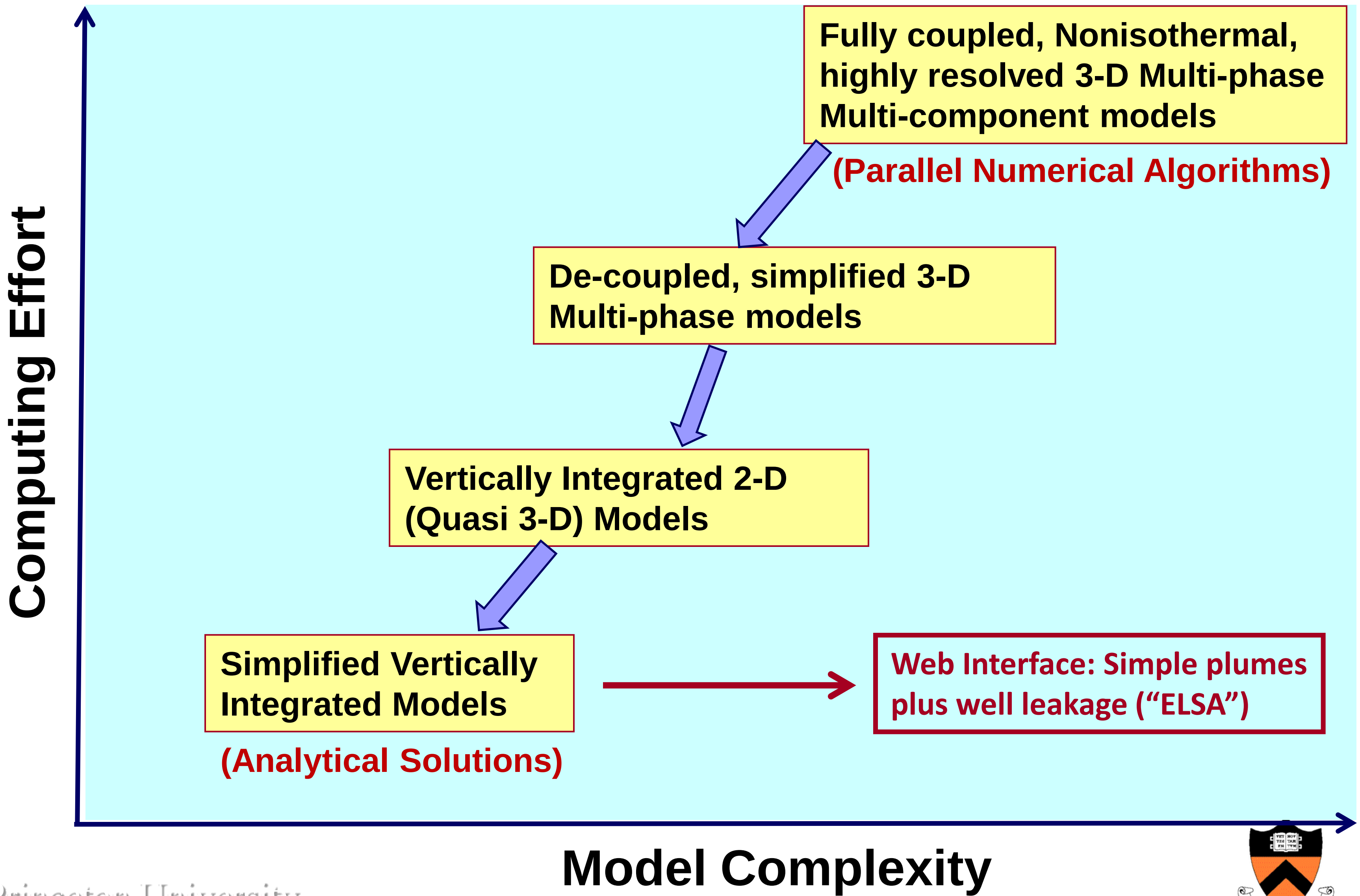
OUTLINE

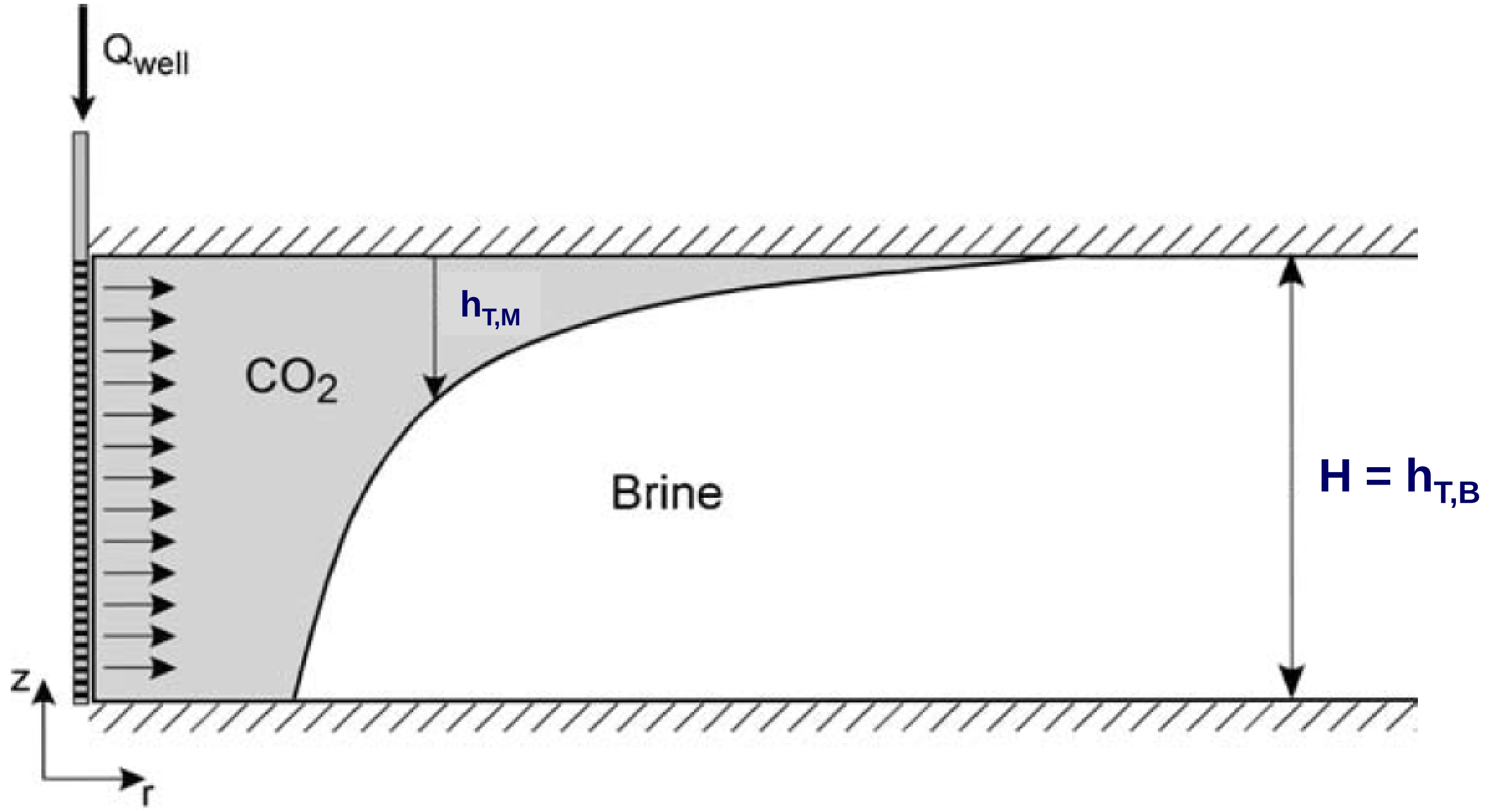
- Model Complexity and Practical Models
- Simple Vertical Equilibrium Models
- Multi-scale Models
  - Vertical Integration and Vertical Reconstruction
  - Dissolution and Convective Mixing
  - Concentrated features / leakage pathways
- Example Calculations
- Final Remarks

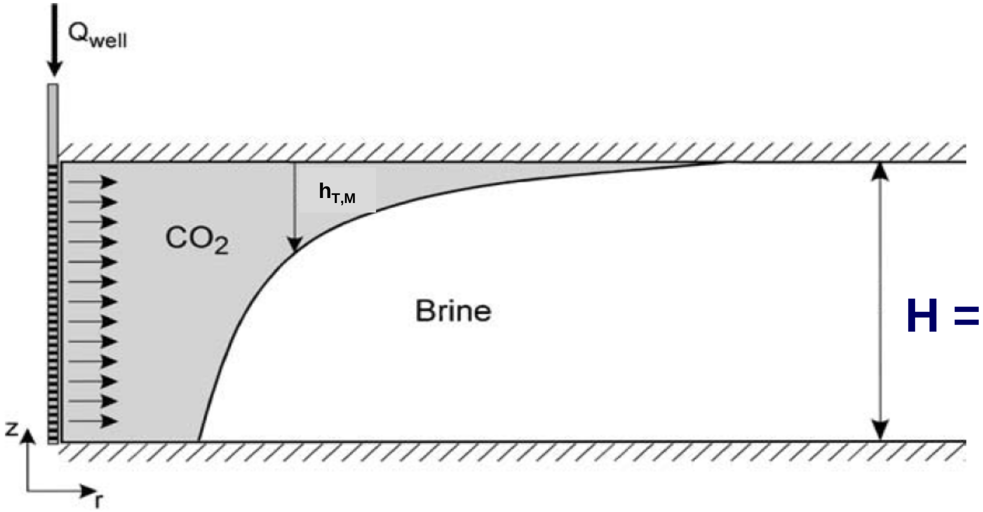
A HIERARCHY OF MODELS



SIMPLEST ANALYTICAL SOLUTION

- Horizontal, homogeneous formations.
- Constant injection rate in a single vertical well.
- Vertical Equilibrium and Sharp Interface



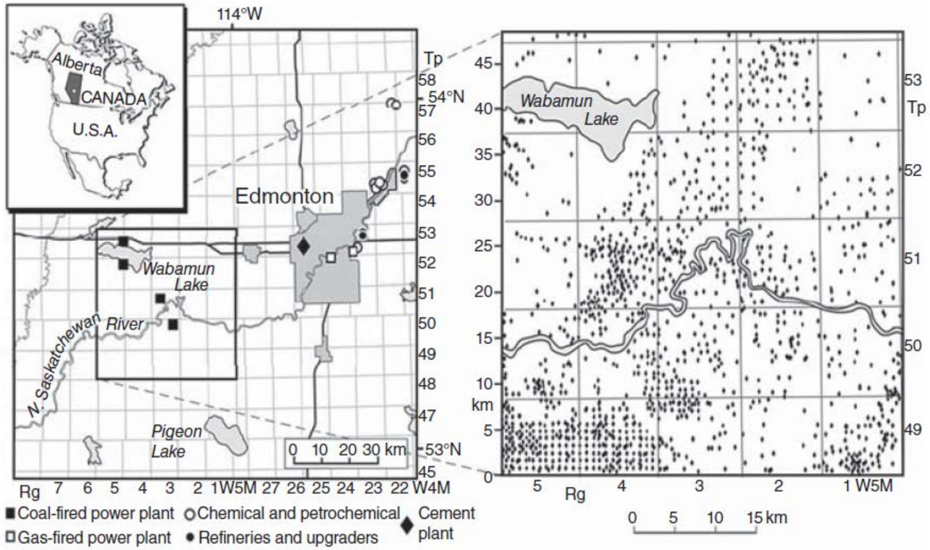


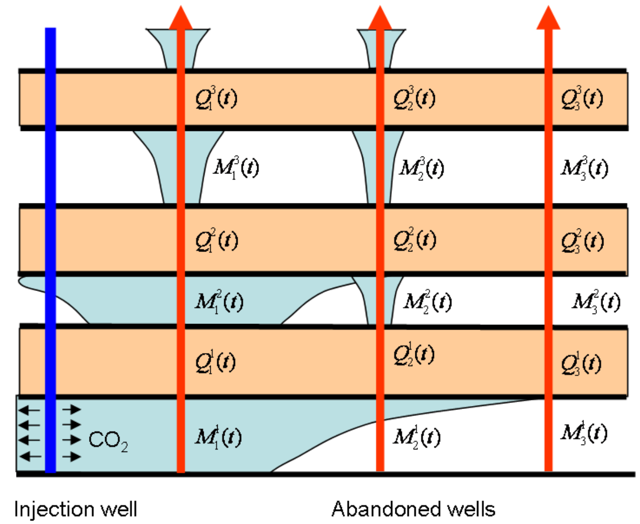
$H = h_{T,B}$

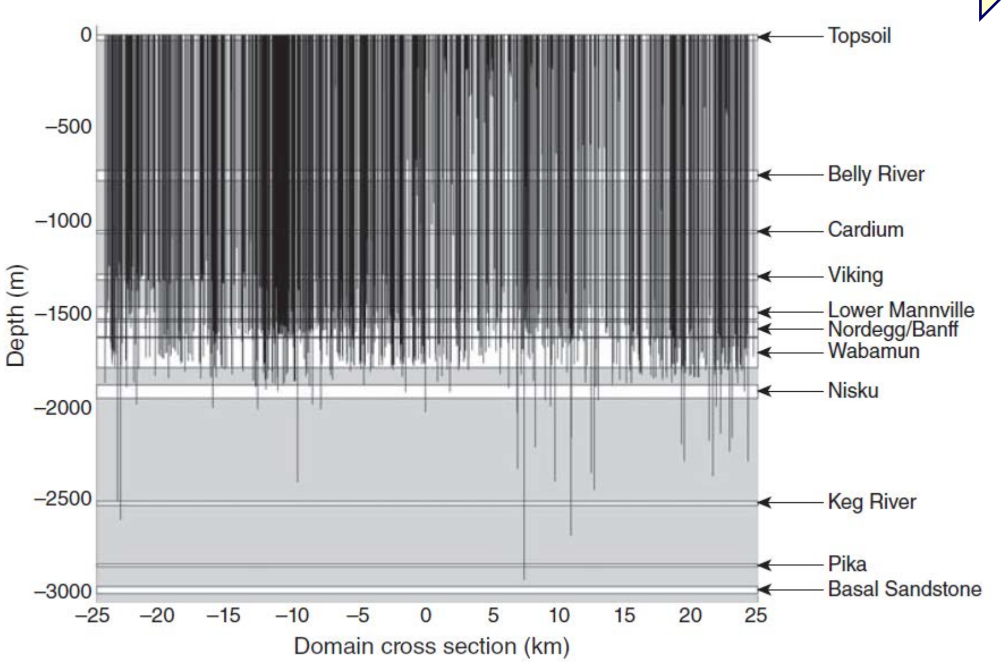
$$\chi \equiv \frac{\pi \Phi r^2}{Q_\Sigma t}$$

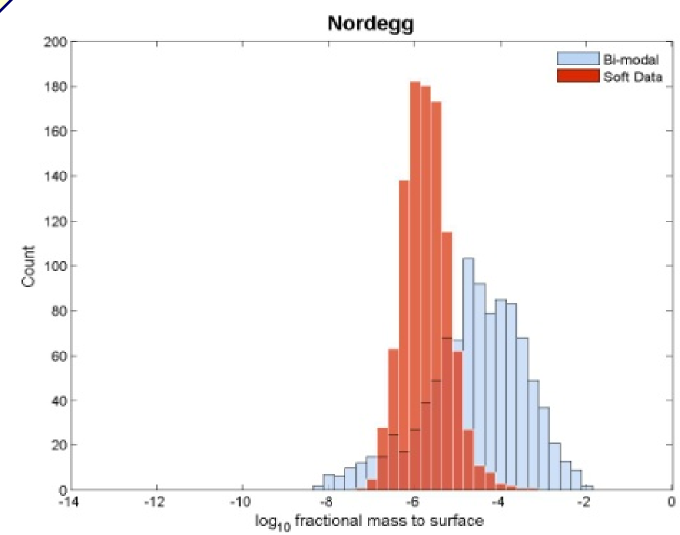
$$\frac{S_c(\chi)}{s_{c,T}} = \frac{h_{T,M}}{h_{T,B}} = \begin{cases} 0 & \text{if} \quad s_{c,T}\chi \geq \lambda \\ \dfrac{1}{\lambda - 1}\left(\sqrt{\dfrac{\lambda}{s_{c,T}\chi}} - 1\right) & \text{if} \quad \lambda^{-1} < s_{c,T}\chi < \lambda \\ 1 & \text{if} \quad 0 < s_{c,T}\chi \leq \lambda^{-1} \end{cases}$$

$$\Gamma = 2\frac{\pi K \Lambda_b' \Delta_\alpha \rho g h_{T,B}}{Q_\Sigma} \quad (< 0.1)$$



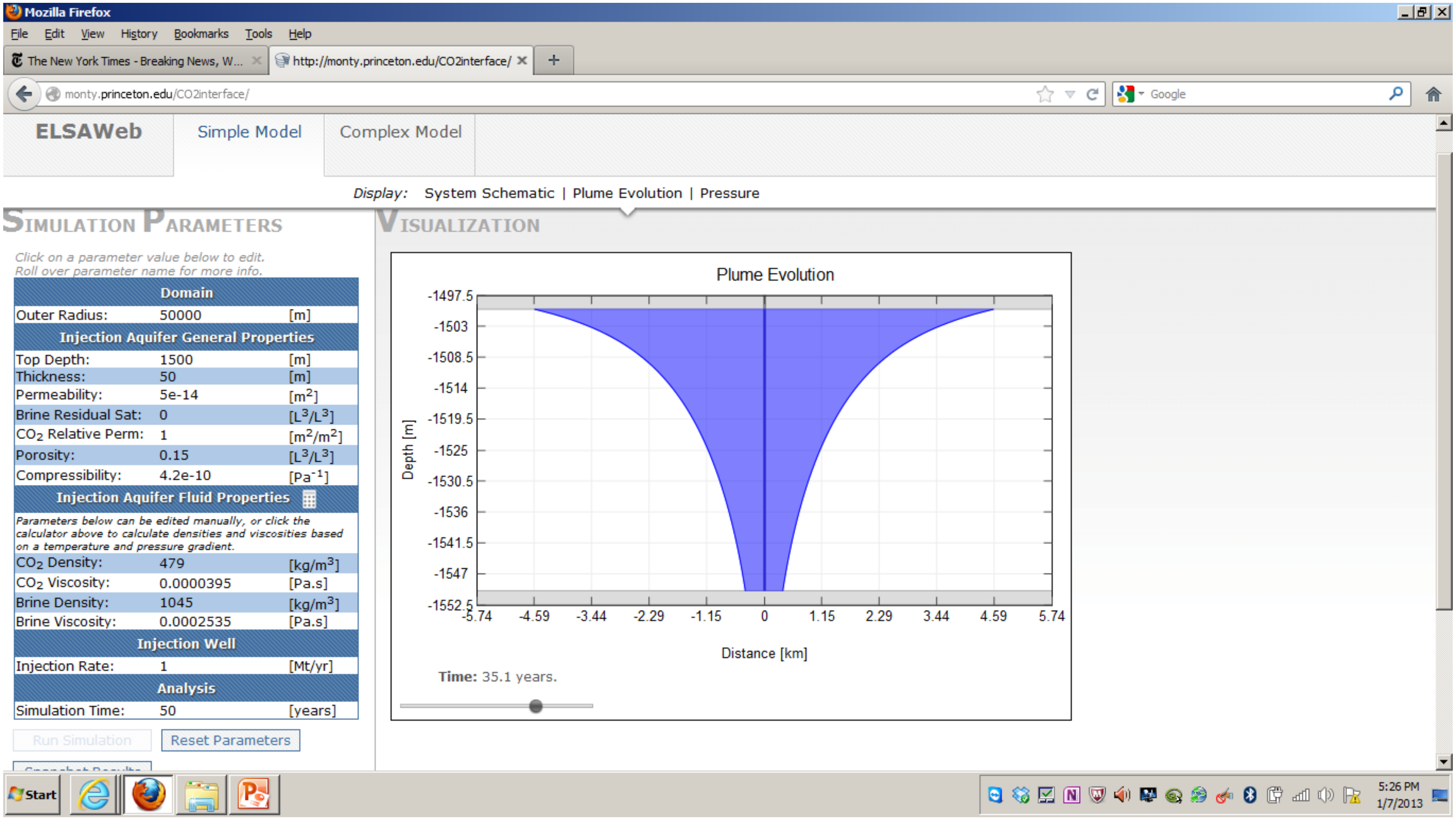






(See: Celia at al., 2011)

MONTY.PRINCETON.EDU/CO2INTERFACE



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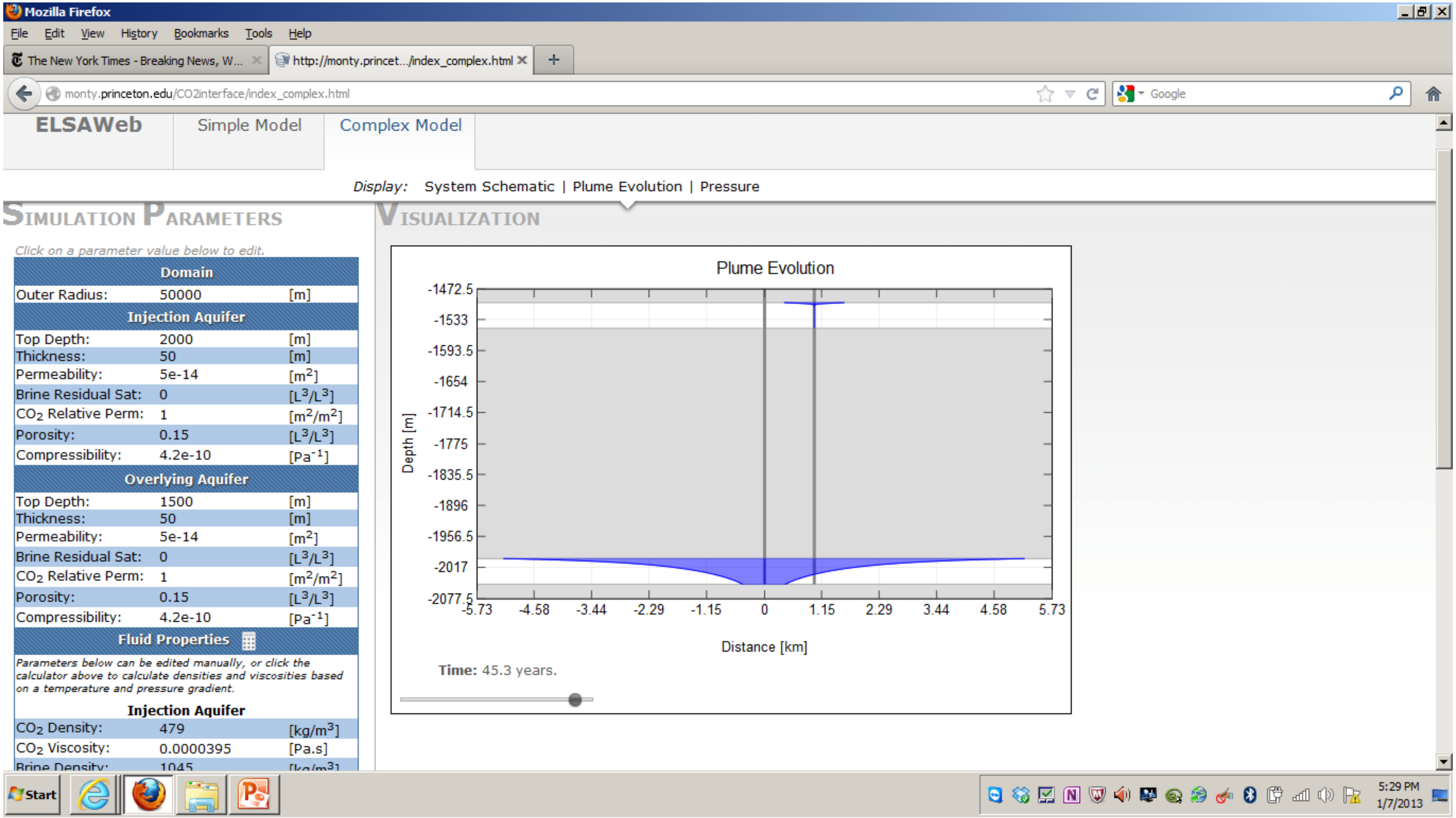
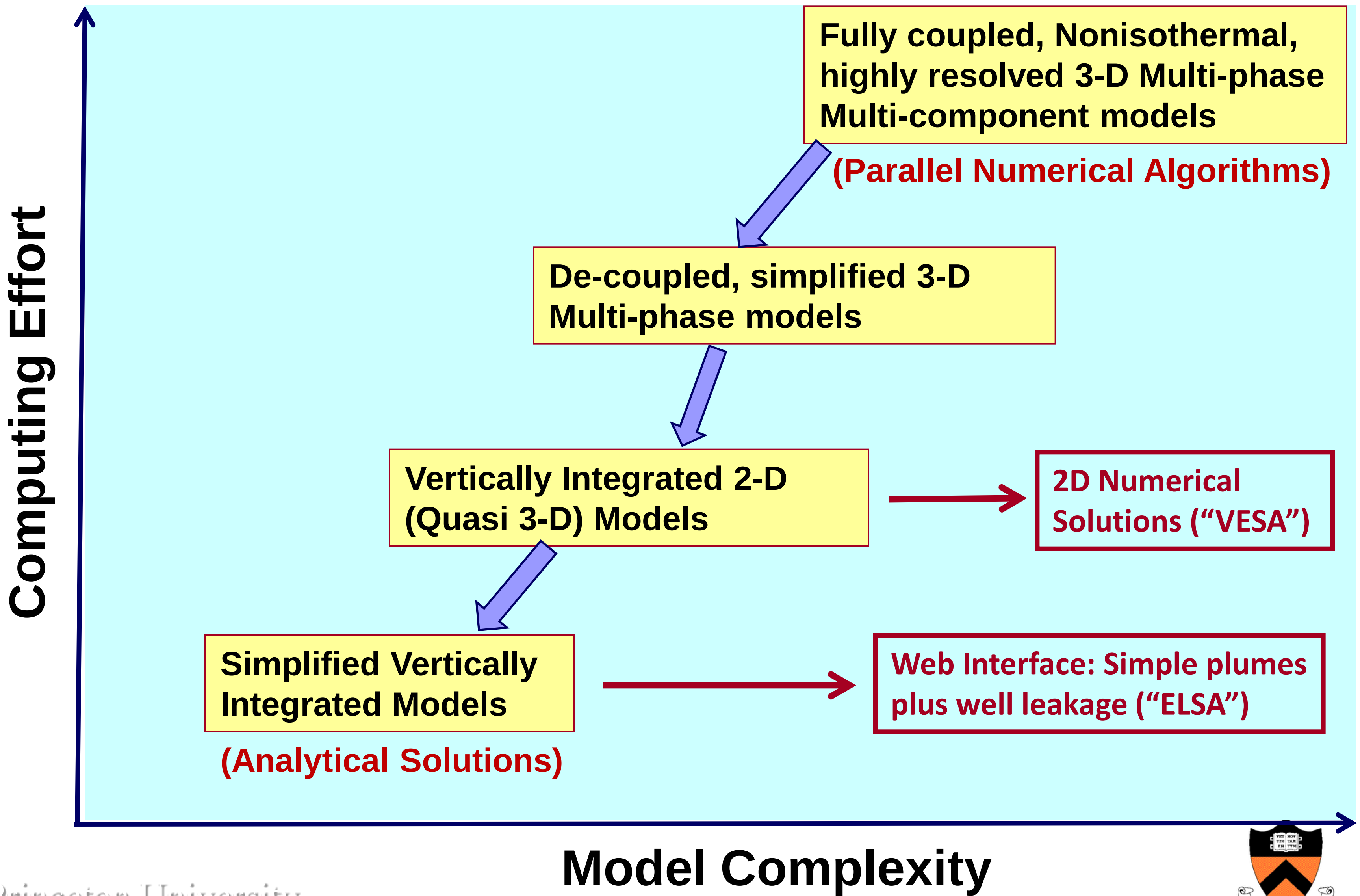
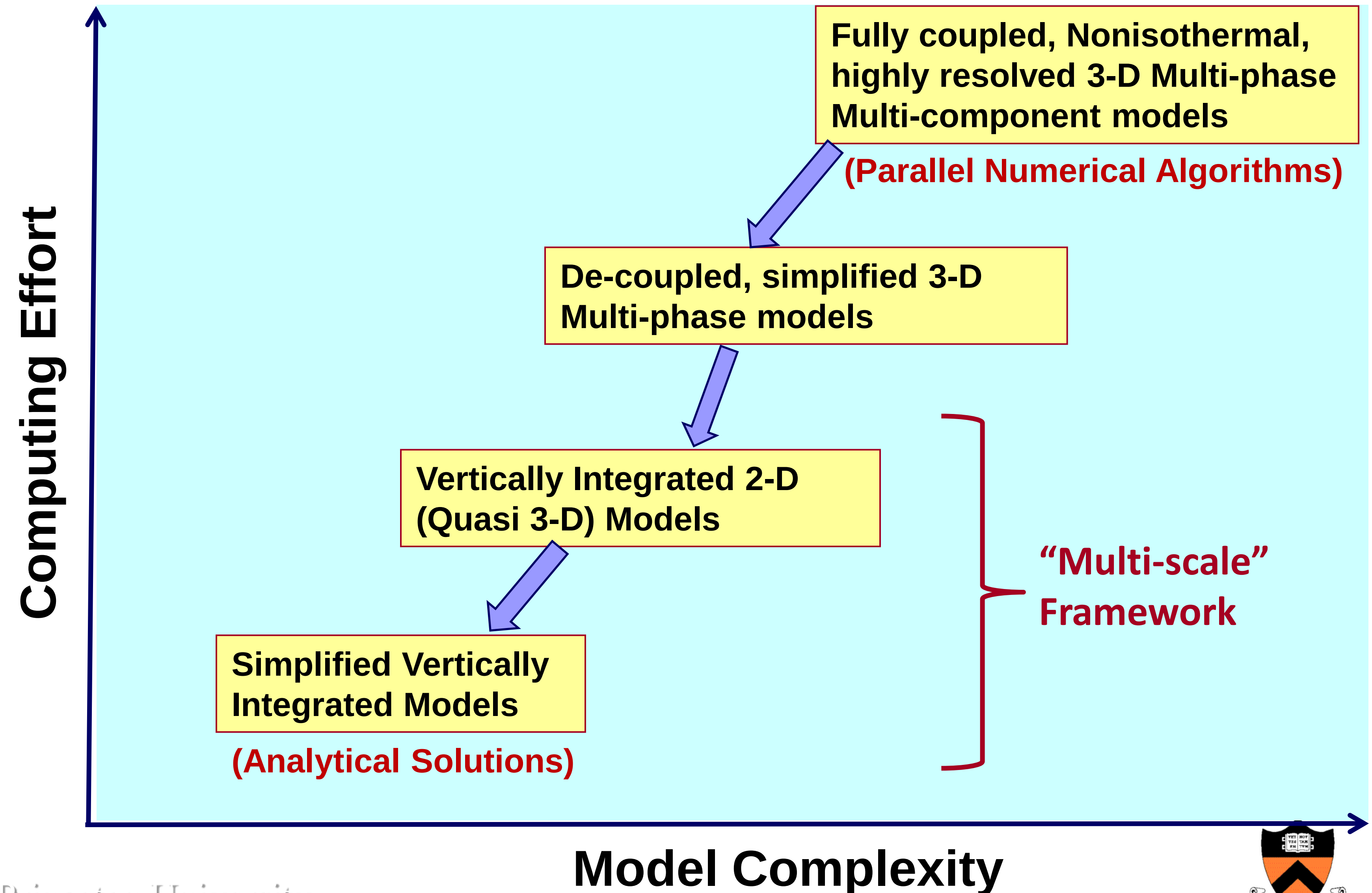


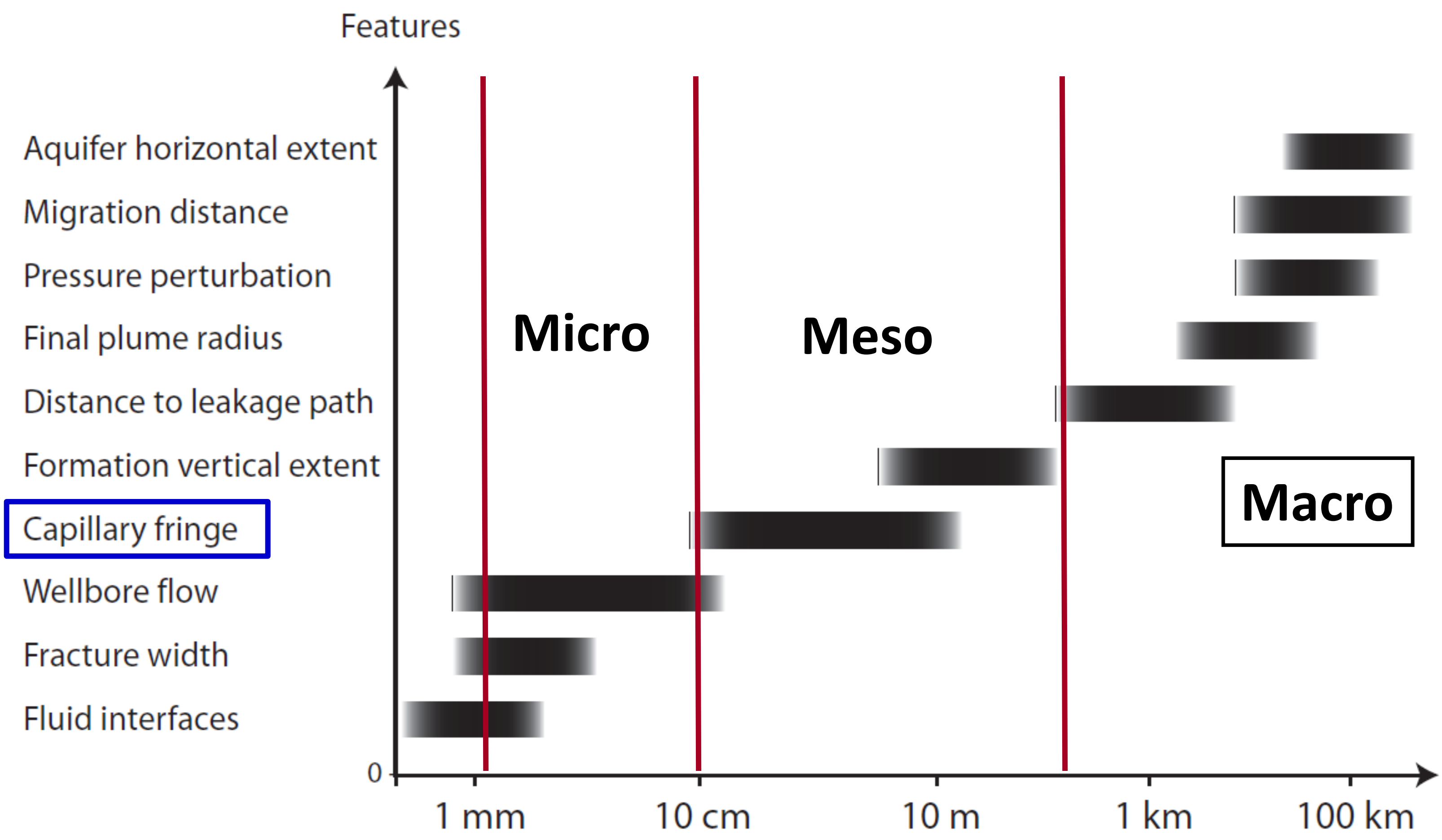
A HIERARCHY OF MODELS



A HIERARCHY OF MODELS

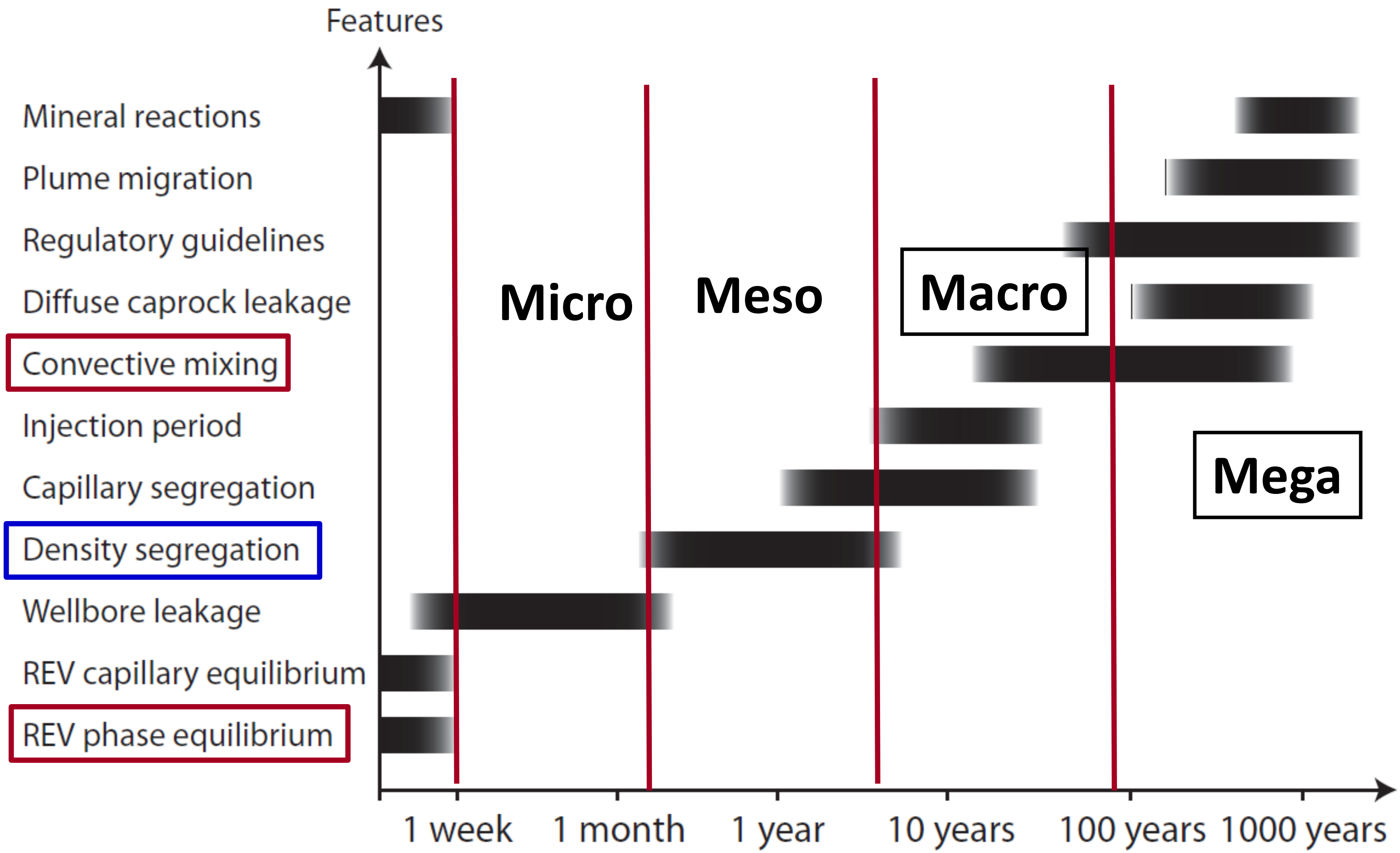


Spatial Scales



(See: Nordbotten and Celia, 2012)

Temporal Scales



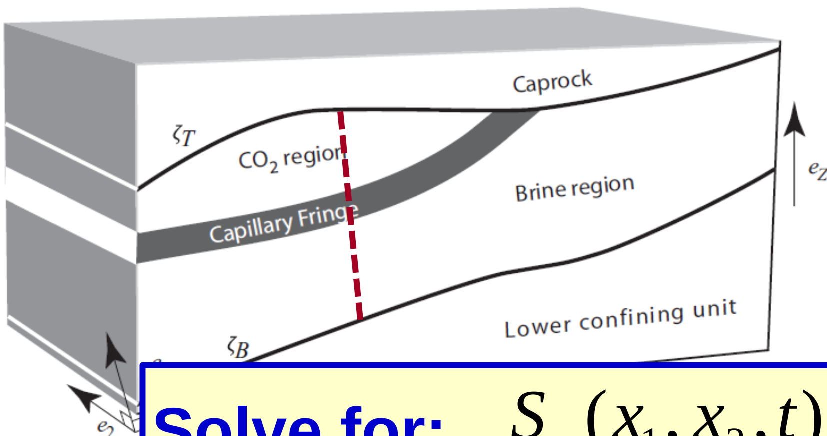
(See: Nordbotten and Celia, 2012)

Vertically Integrated Models (VESA)

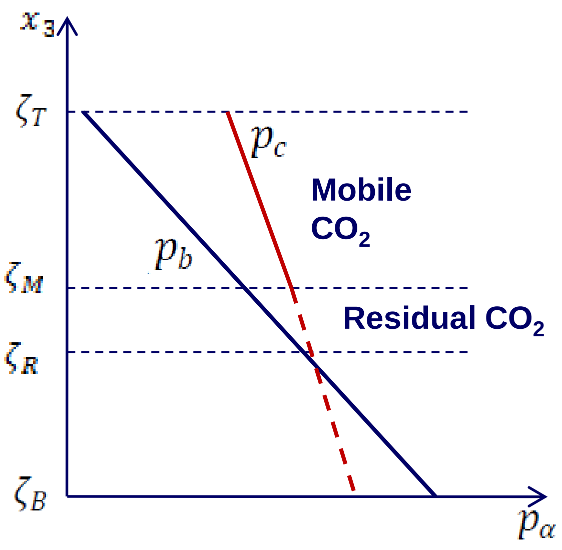
Key Assumption: *Density segregation* has occurred ($t^* << T$).

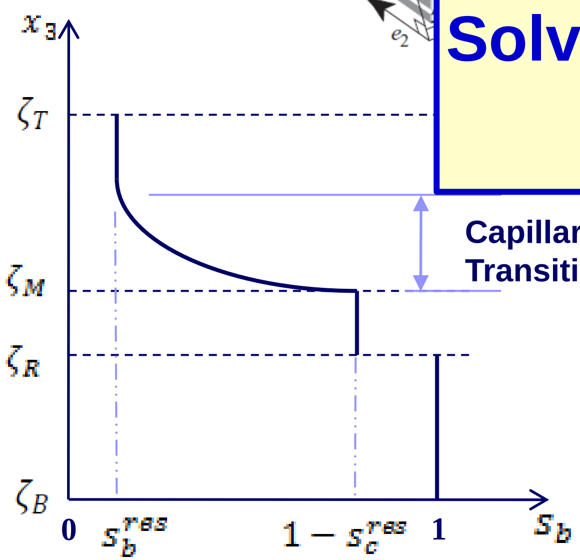
Simplest Case: Zero velocity along x_3 (“vertical equilibrium” or “Dupuit assumption”)

$$\boxed{t^* \sim f(\Delta\rho, k_z, H, k^*_{rel})}$$



Solve for: $S_\alpha(x_1, x_2, t)$
$P_\alpha(x_1, x_2, t)$





Vertically Integrated Models

$$\Phi \frac{\partial S_\alpha}{\partial t} - \nabla_{||} \cdot \left(\boldsymbol{K} \boldsymbol{\Lambda}_\alpha (\nabla_{||} P_\alpha - \varrho_\alpha \boldsymbol{G}) \right) = \Upsilon_\alpha$$

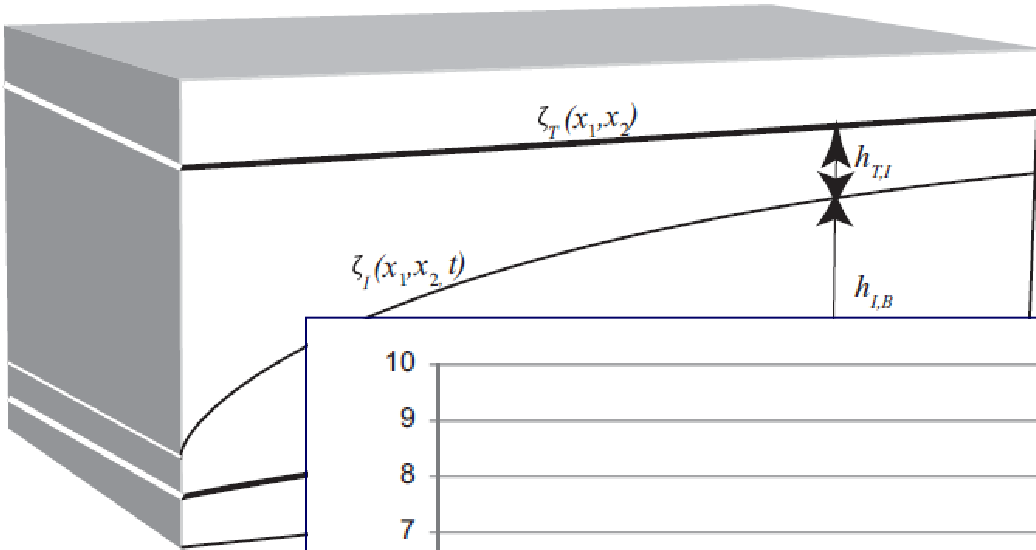
$$\Phi = \int_{\zeta_B}^{\zeta_T} \varphi \, dx_3 \qquad S_\alpha = \frac{1}{\Phi} \int_{\zeta_B}^{\zeta_T} \varphi s_\alpha dx_3$$

$$\boldsymbol{K} = \int_{\zeta_B}^{\zeta_T} \boldsymbol{k}_{||} dx_3 \qquad \boldsymbol{\Lambda}_\alpha = \boldsymbol{K}^{-1} \int_{\zeta_B}^{\zeta_T} \boldsymbol{k}_{||} \boldsymbol{\lambda}_{\alpha,||} dx_3 \qquad \boldsymbol{G} = \boldsymbol{e}_{||} \cdot \boldsymbol{g} + (\boldsymbol{g} \cdot \boldsymbol{e}_3) \nabla_{||} \zeta_P$$

P_α is the phase pressure at a chosen datum elevation ($x_3 = \zeta_P$)

1. **Solve coarse equation for** $S_\alpha(x_1, x_2, t), P_\alpha(x_1, x_2, t)$
2. **Reconstruct** $\hat{s}_\alpha(x_1, x_2, x_3, t),\ \hat{p}_\alpha(x_1, x_2, x_3, t)$
3. **Update mobilities** $\hat{\lambda}_\alpha(x_1, x_2, x_3, t),\ \Lambda_\alpha(x_1, x_2, x_3, t)$

Sharp Interface Assumption



Capillary Transition Zone negligible

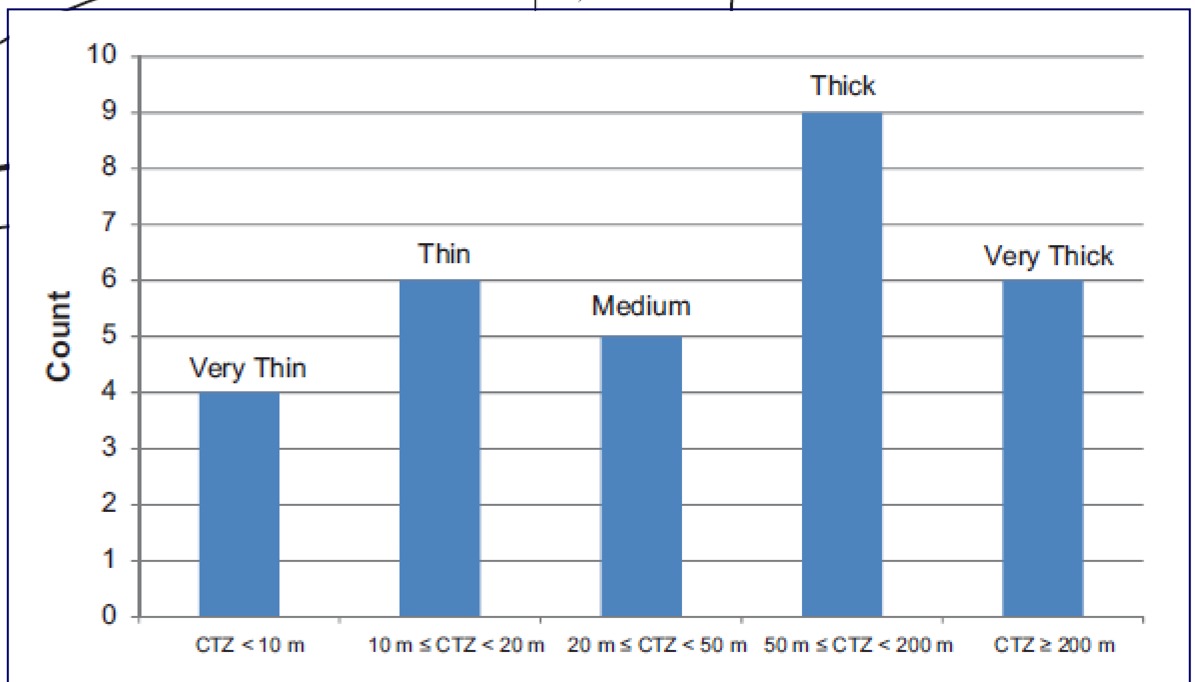
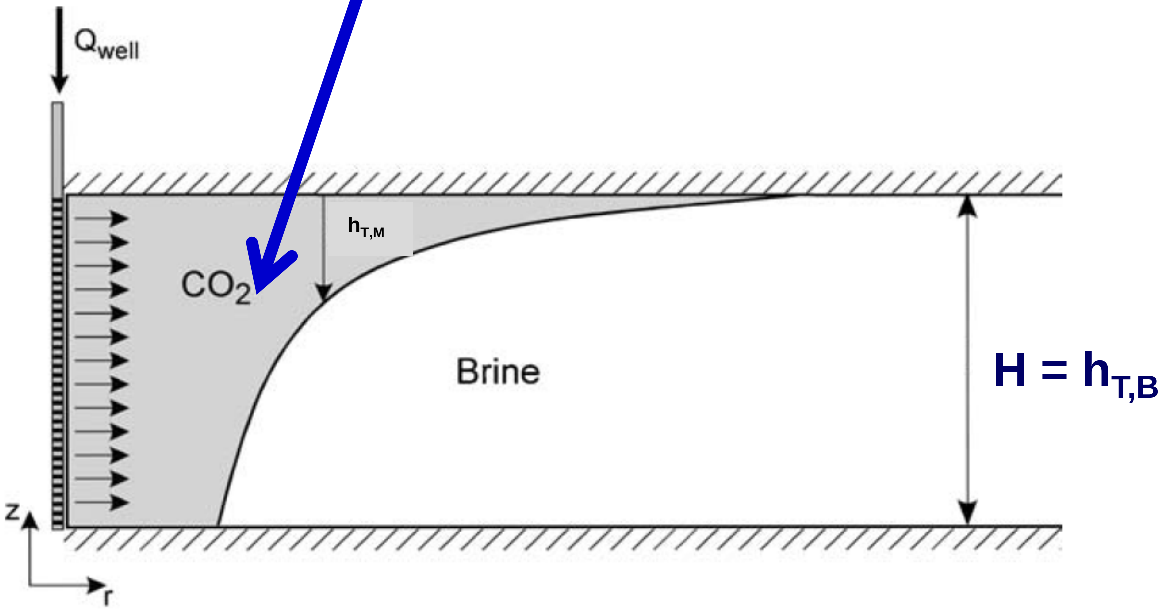


Fig. 3. Capillary Transition Zone (CTZ) thicknesses reported in Table 2 grouped into five typical aquifer thickness categories.

(See: Court et al., 2012)

Dissolution and Geochemistry

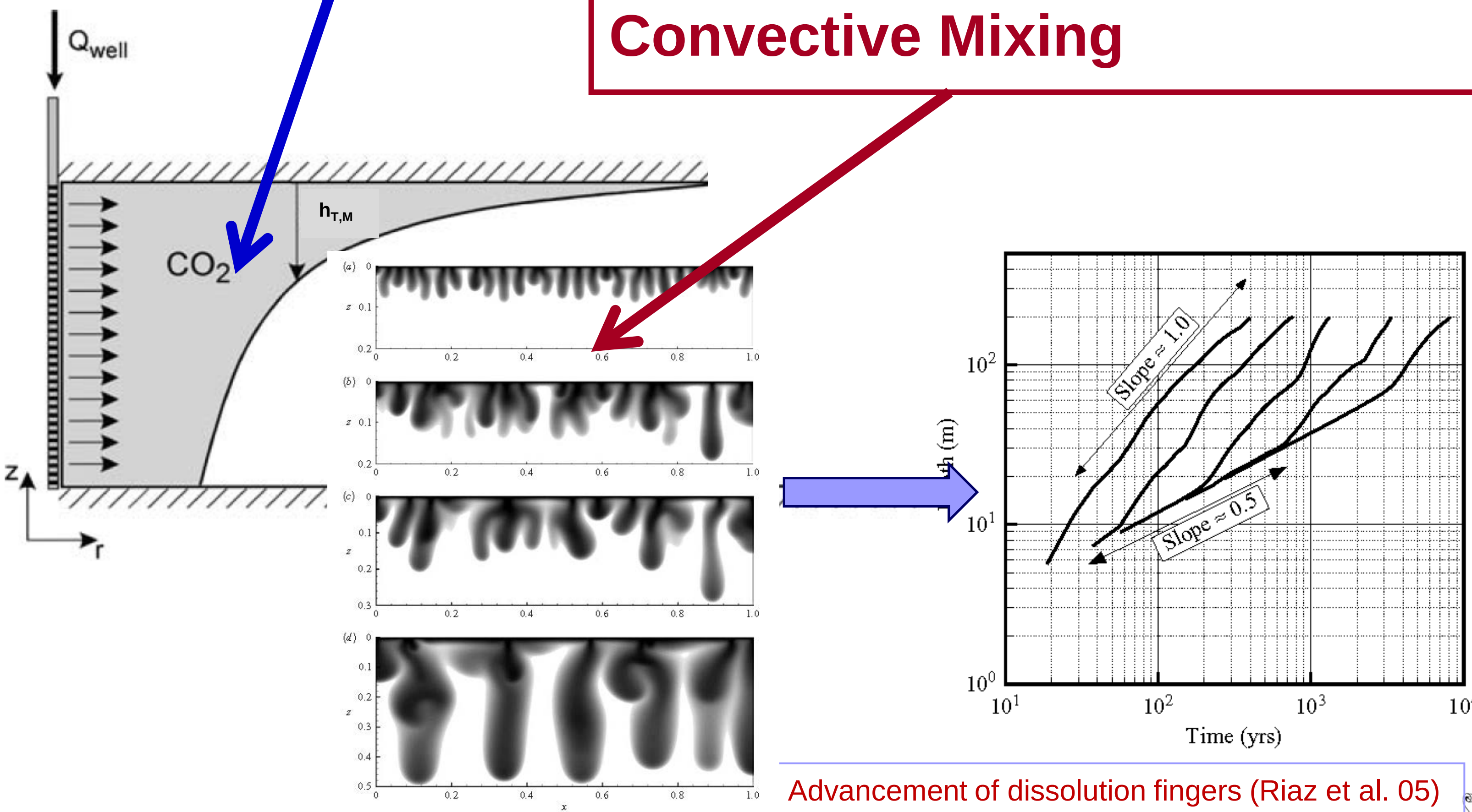
CO_2 Plume: Equilibrium Partitioning



Dissolution and Geochemistry

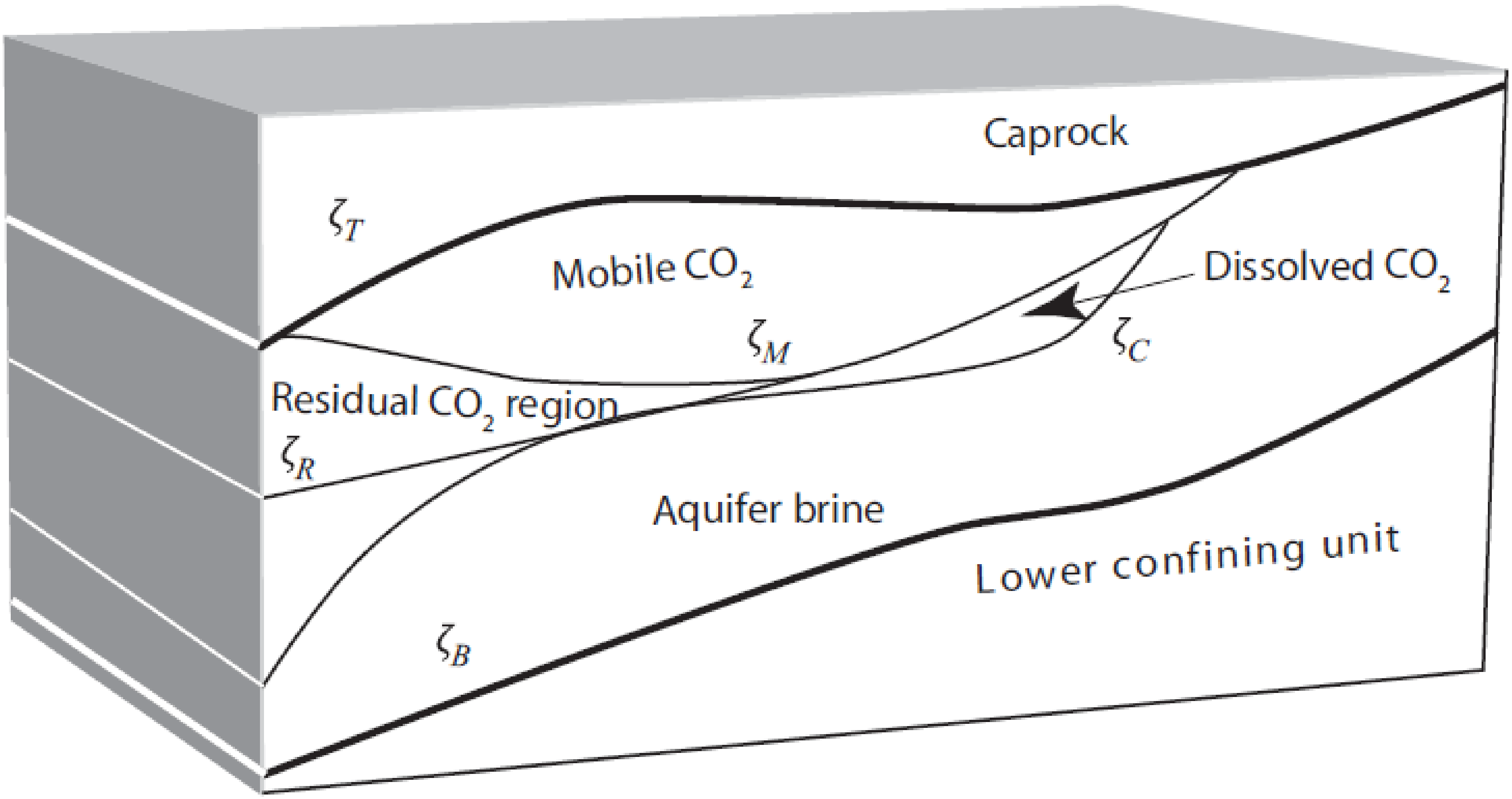
CO_2 Plume: Equilibrium Partitioning

Dissolved CO_2 Transport in Brine: Convective Mixing



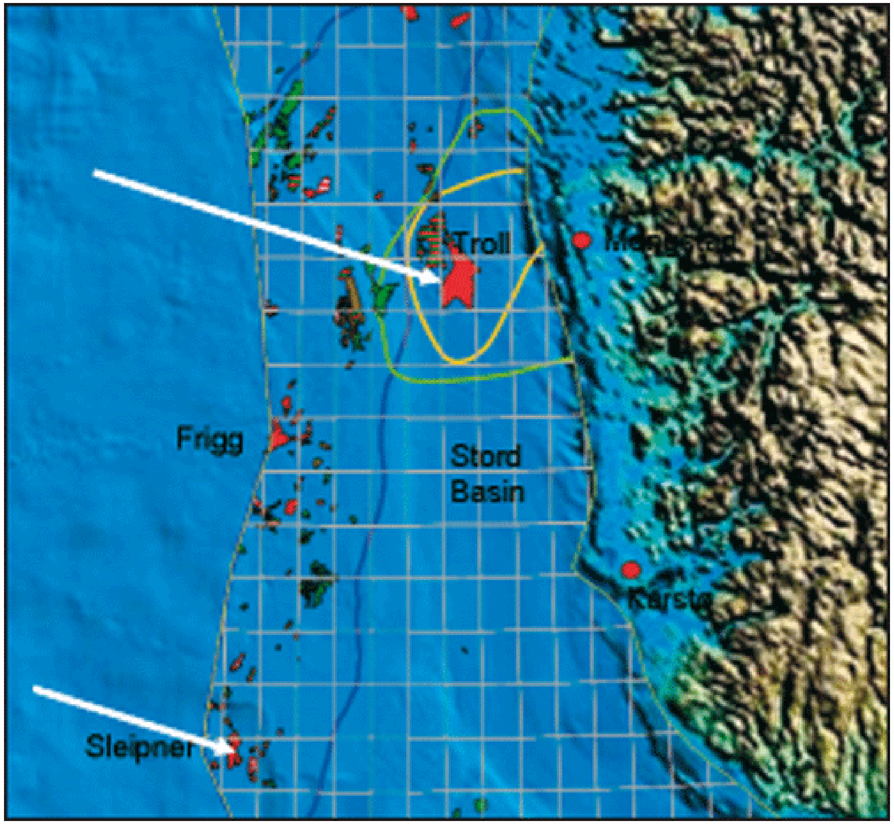
Advancement of dissolution fingers (Riaz et al. 05)

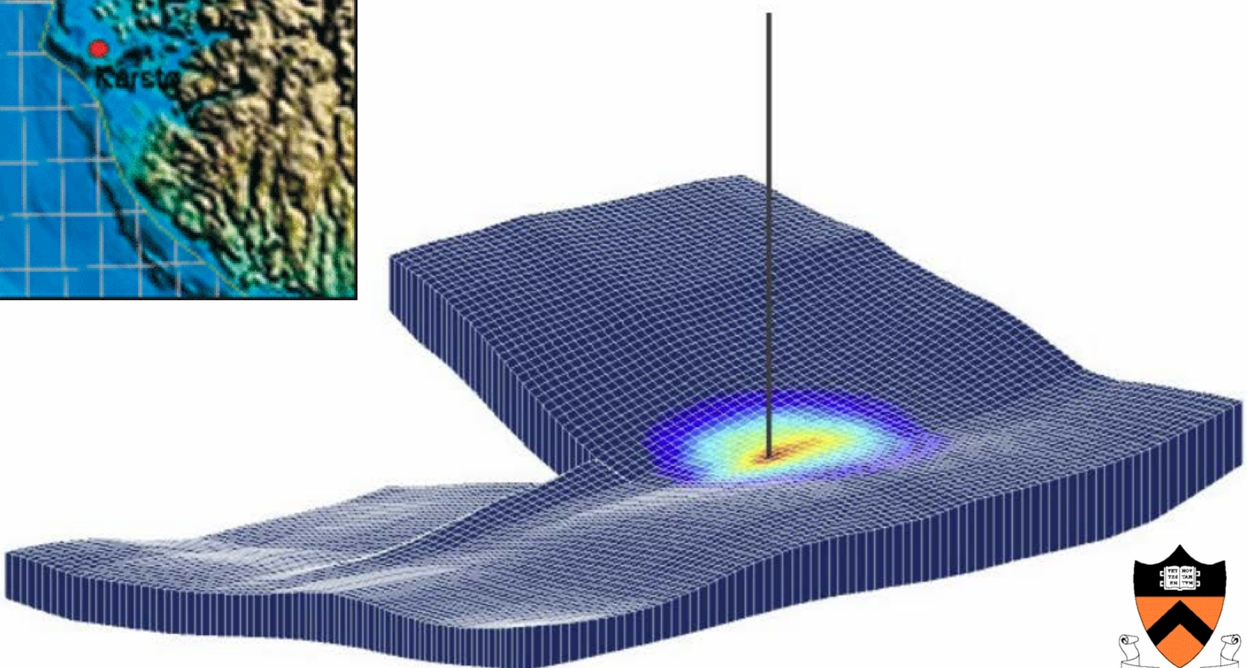
MODELS WITH CAPILLARY TRAPPING AND DISSOLUTION

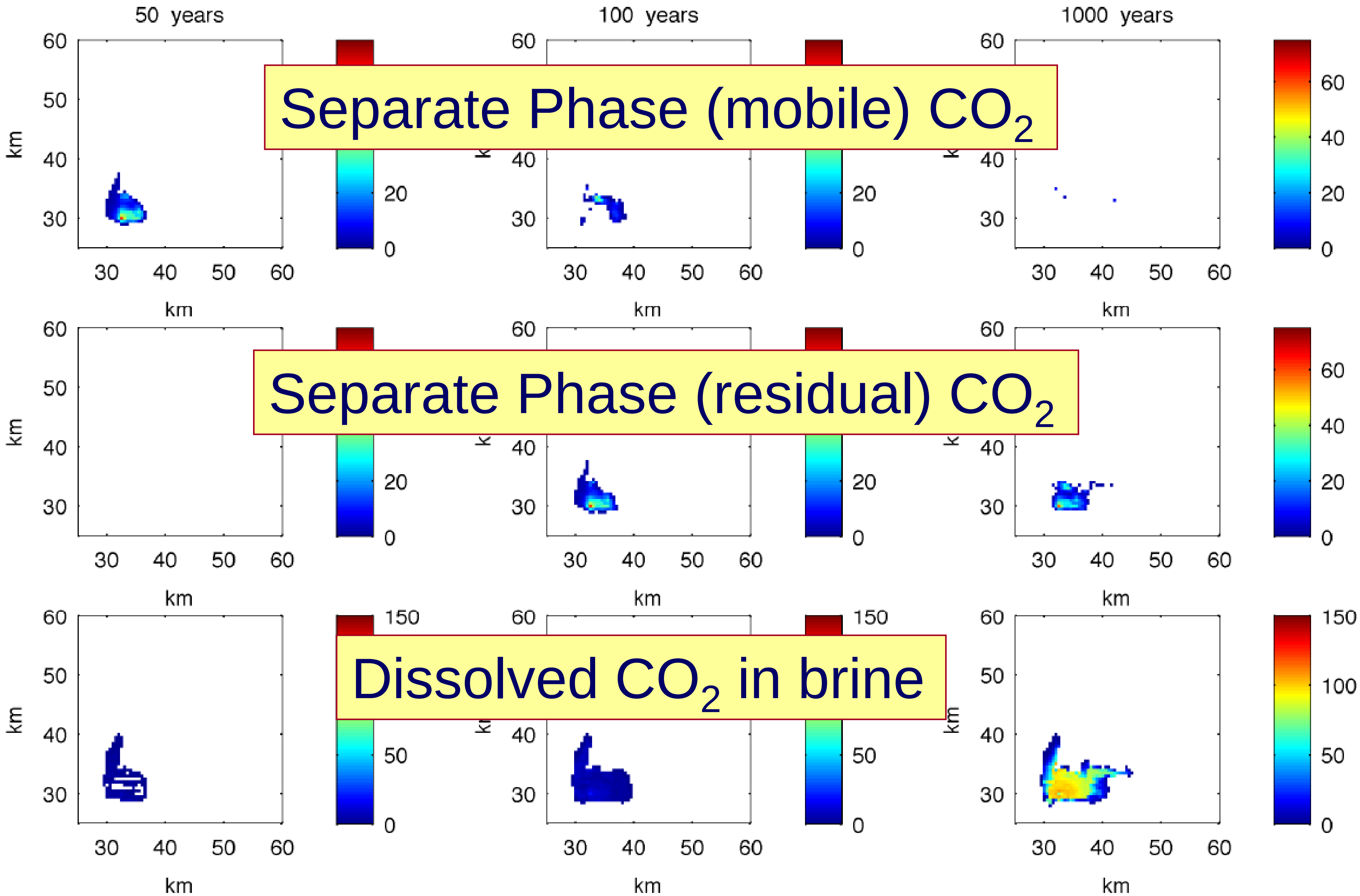


(See: Bandilla et al., 2013; Gasda et al., 2011, 2012; Nordbotten and Celia, 2012)

LARGE-SCALE APPLICATION OF VE MODEL

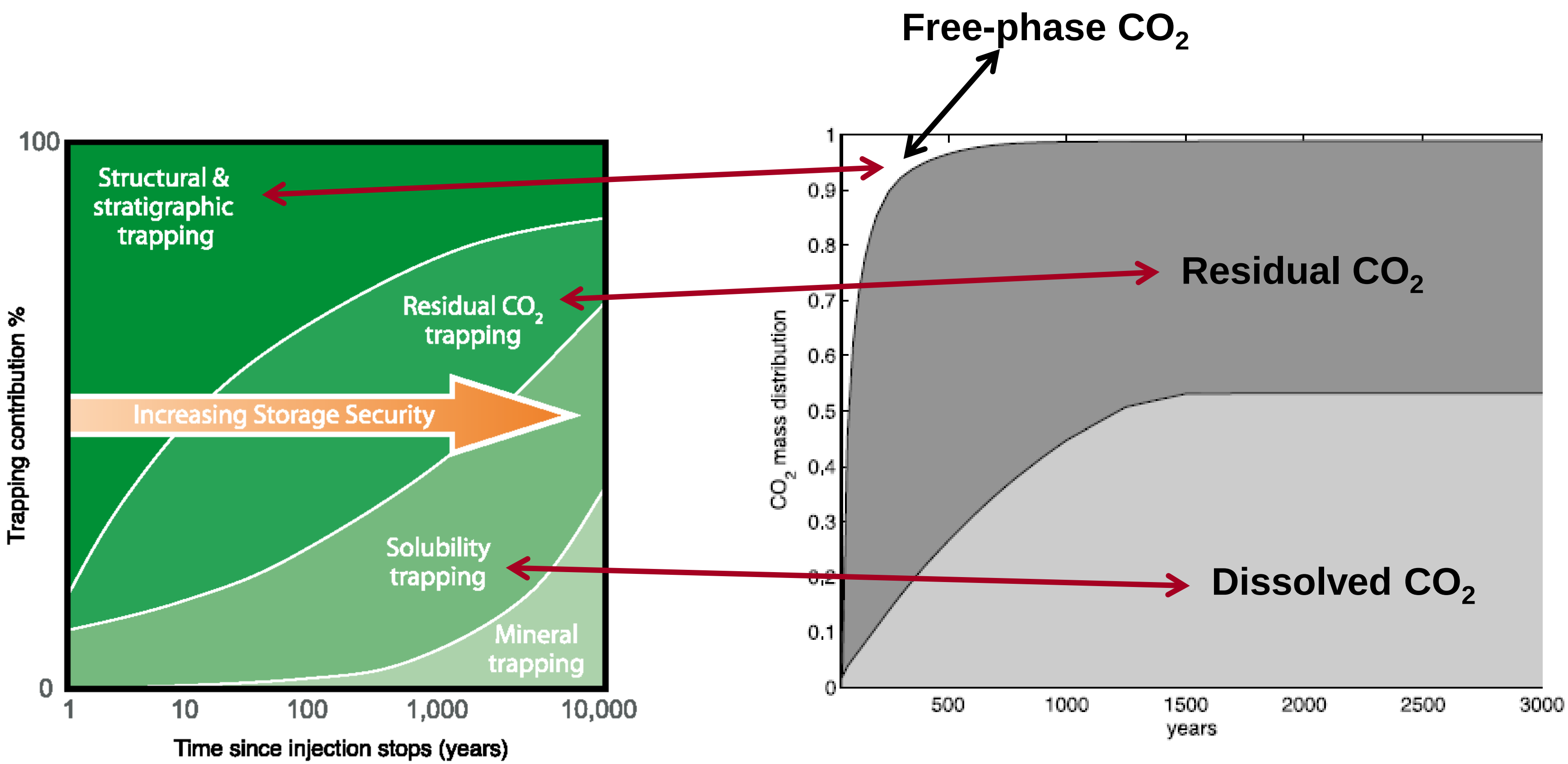




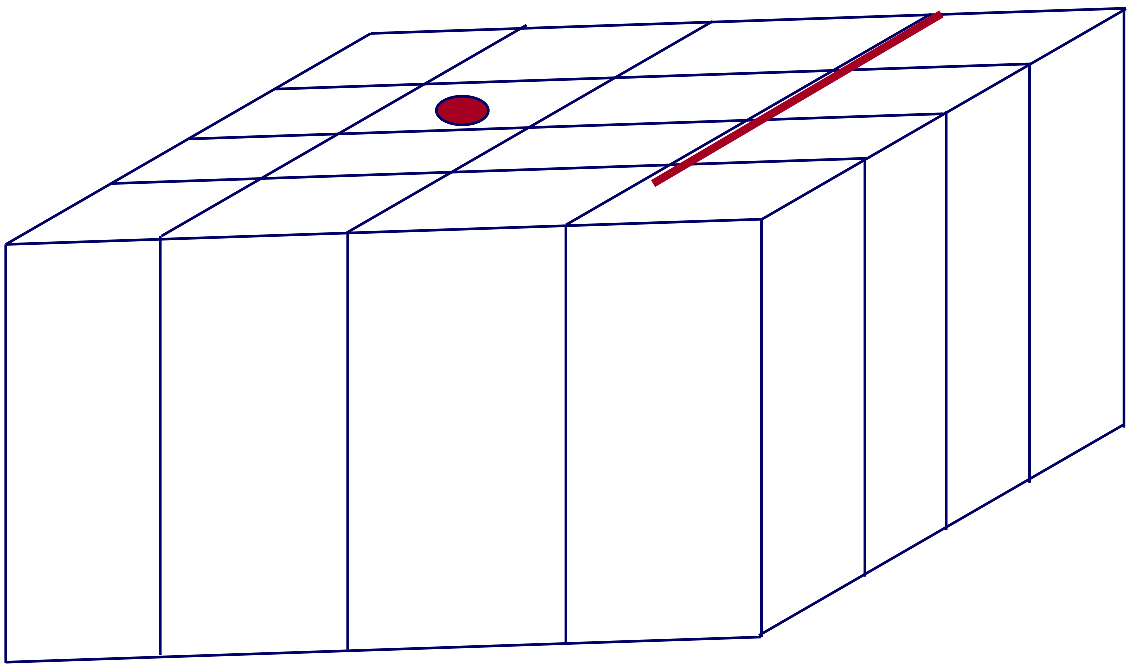


(See: Gasda et al., 2011, 2012)

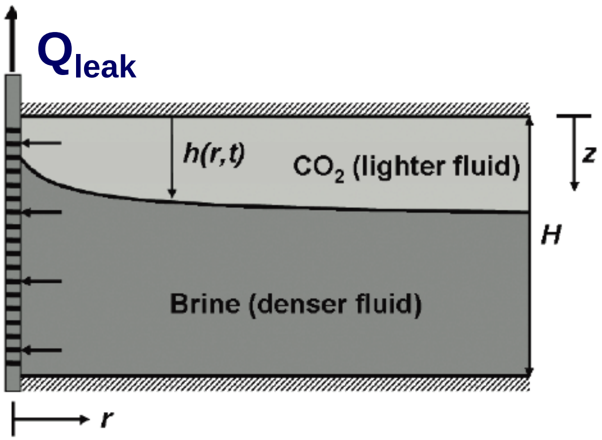
Long-term Predictions



Fault, Fractures, Wells, Leakage



Modified “Peaceman-type” corrections



$$\Gamma\lambda\frac{dh}{dx} + \frac{k'}{3}\left[-2\Gamma\lambda\frac{\left(\frac{dh}{dx}\right)^3}{1+k'\left(\frac{dh}{dx}\right)^2} + \lambda Q'_d\left(\frac{d^2h}{dx^2}+\frac{1}{1-h}\left(\frac{dh}{dx}\right)^2\right) + Q'_l\left(\frac{d^2h}{dx^2}-\frac{1}{h}\left(\frac{dh}{dx}\right)^2\right)\right] = \frac{Q'_d\lambda}{1-h}-\frac{Q'_l}{h}.$$

CONCLUDING COMMENTS

1. Vertically integrated models are often reasonable choices for practical calculations

2. A Multi-scale framework allows assumptions and representations across scales to be identified explicitly.

3. Most important processes can be included.

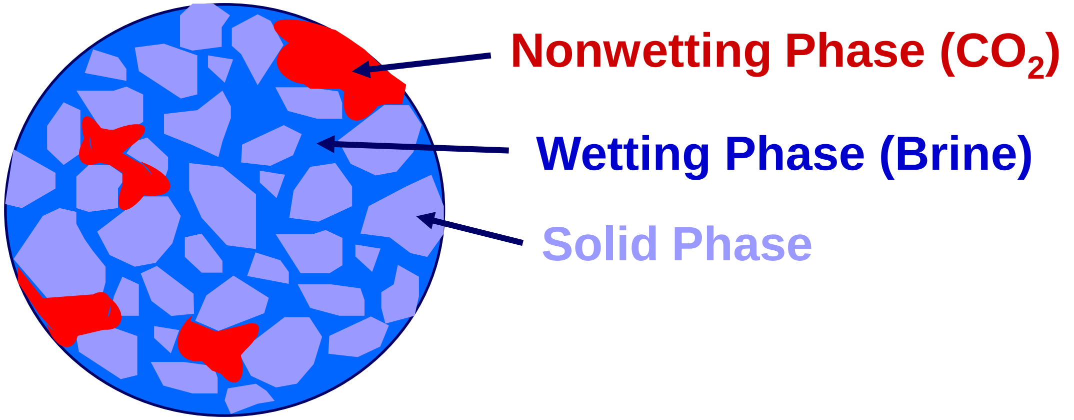
4. Extensions include complete hysteresis, thermal effects, geomechanics, and vertical dynamics.

5. Models should be compatible with the questions being asked and the available data.

MULTI-SCALE MODELS

Upscaling ⟷ Downscaling

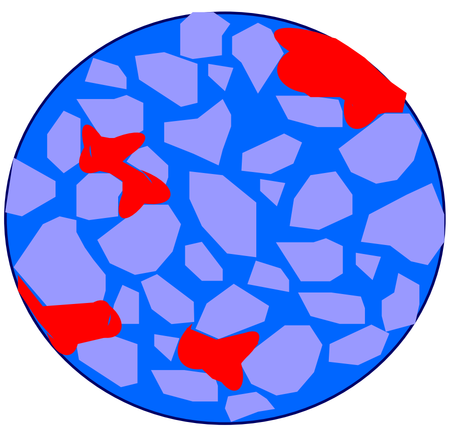
Compression ⟷ Reconstruction



MULTI-SCALE MODELS

Upscaling $\longleftrightarrow$ Downscaling

Compression $\longleftrightarrow$ Reconstruction



Compression $\longrightarrow$

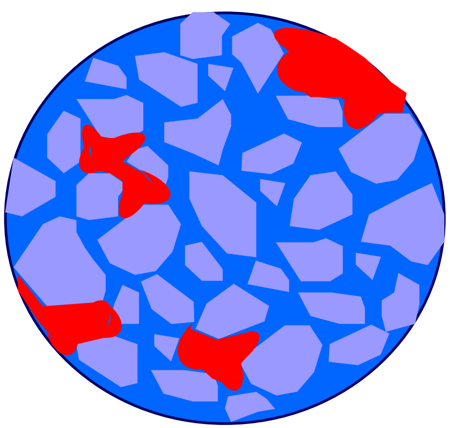
$$\varphi = \frac{V_{voids}}{V_{tot}}$$

$$s_\alpha = \frac{V_\alpha}{V_{voids}}$$

MULTI-SCALE MODELS

Upscaling $\longleftrightarrow$ Downscaling

Compression $\longleftrightarrow$ Reconstruction



Reconstruction

$\longleftarrow$

??

$$\varphi = \frac{V_{voids}}{V_{tot}}$$

$$s_{\alpha} = \frac{V_{\alpha}}{V_{voids}}$$