# Evaluation of Standards data collected from probabilistic sampling programs

Eric P. Smith

Y. Duan, Z. Li, K. Ye

Statistics Dept., Virginia Tech

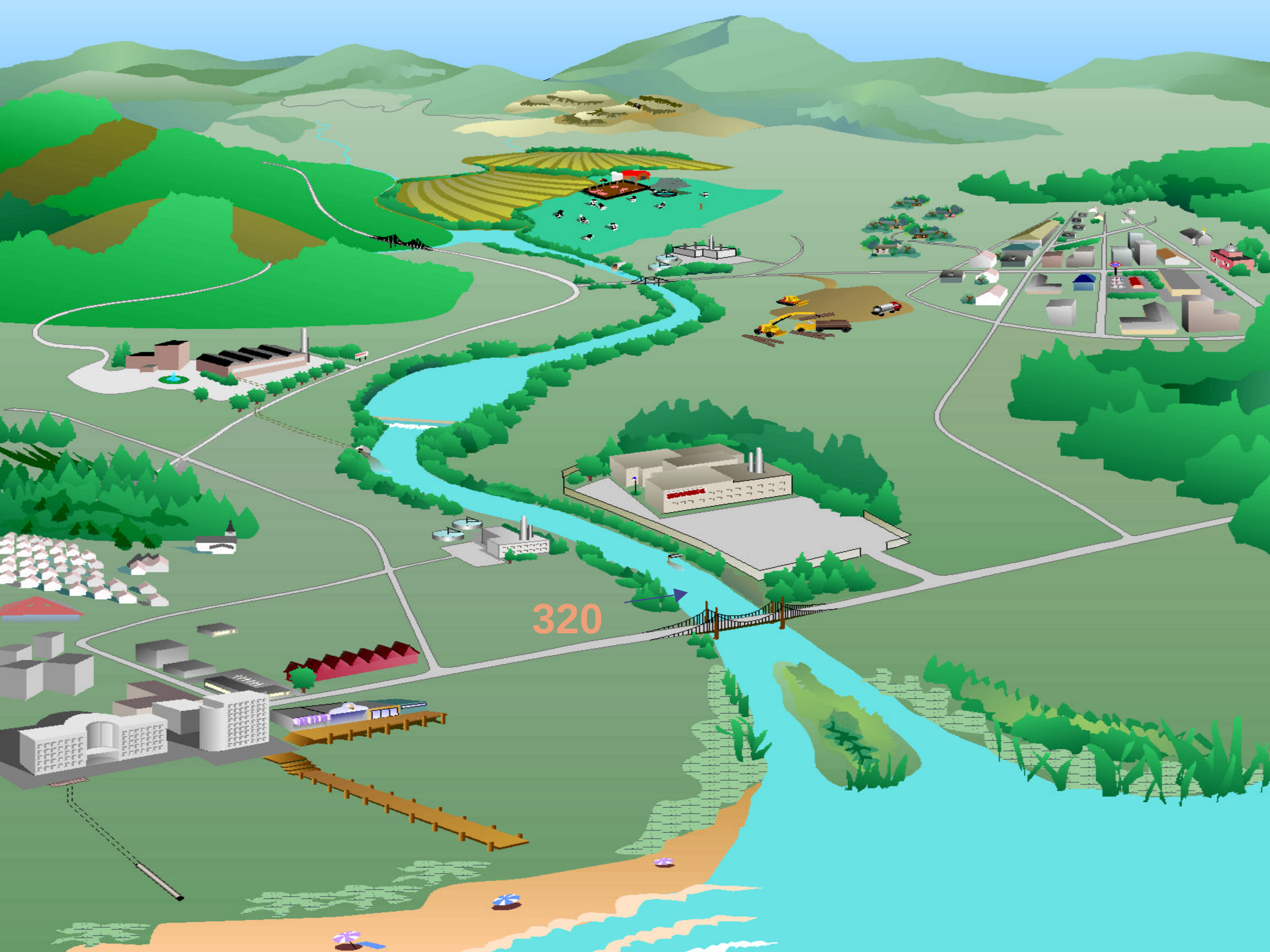
# Sponsor

This research is funded by
U.S. EPA - Science To Achieve Results (STAR) Program
Grant # RD-83136801-0

This talk was not subjected to USEPA review. The conclusion and opinions are soley those of the authors and not the views of the Agency.

# Outline

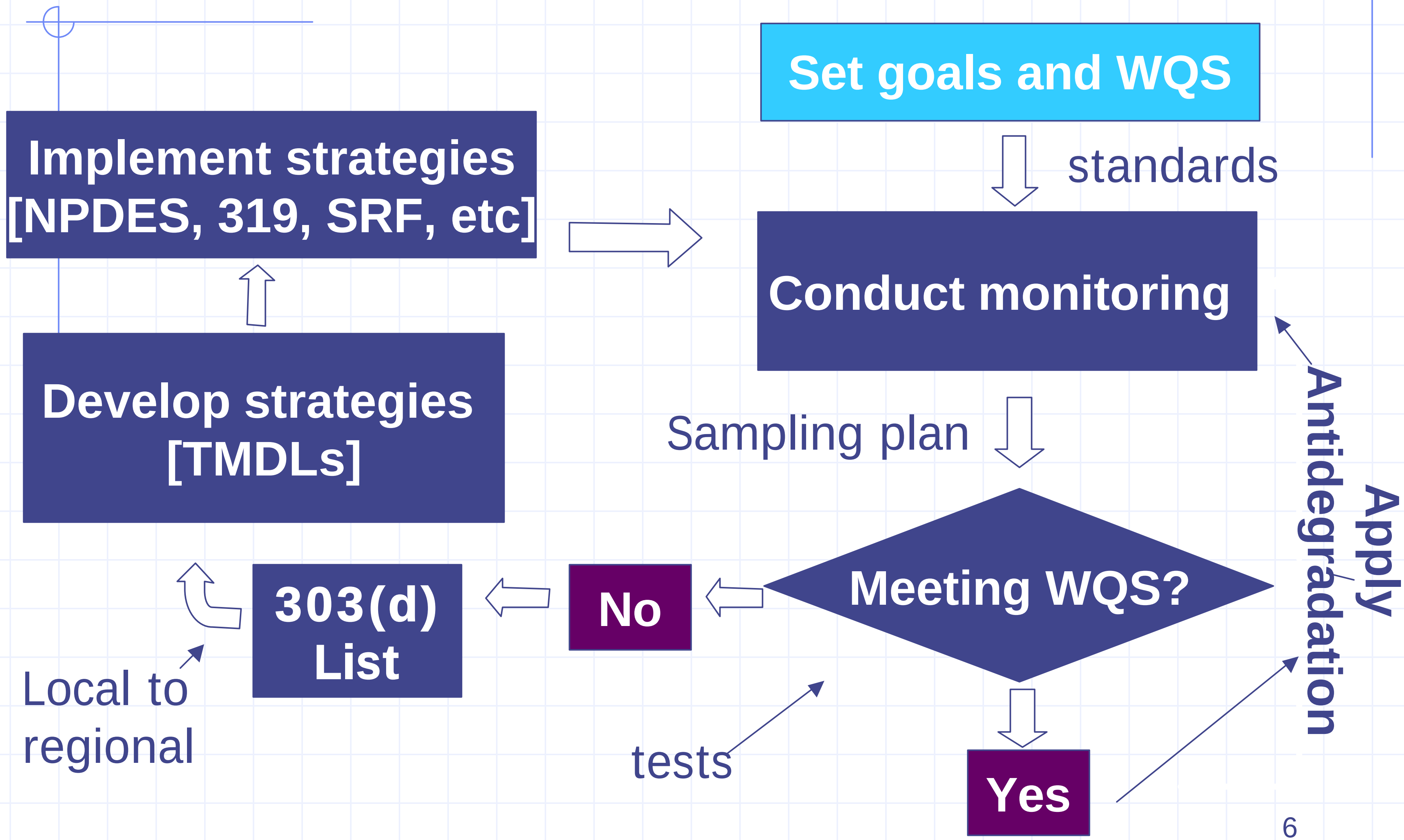
- Background
  - Standards assessments
- Single site analysis
- Regional analysis
  - Mixed model approach
  - Bayesian approach
- Upshot: need models that allow for additional information to be used in assessments


320

# Standards assessment – 303d

- Clean Water Act section 303d mandates states in US to monitor and assess condition of streams
- Site impaired – list site, start TMDL process (Total Max Daily Loading)
- Impaired means site does not meet usability criteria

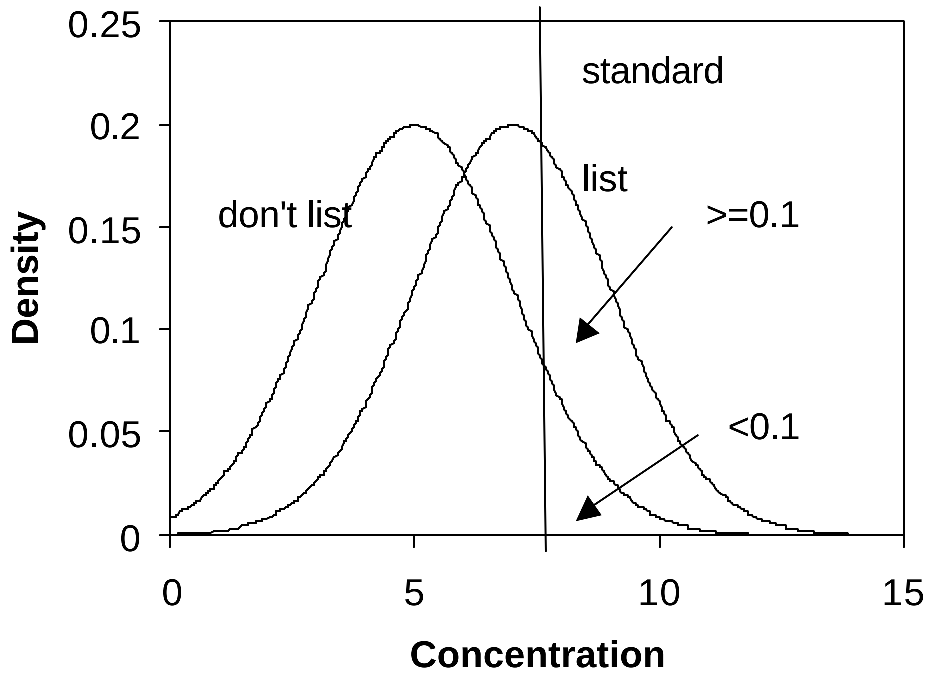
# Linkages in 303(d)



# Impaired sites

- Site impaired if standards not met
- Standards – defined through numerical criteria
  - Involve frequency, duration, magnitude
- –Old method
  - Site impaired if >10% of samples exceed criteria
  - Implicit statistical decision process- error rates

# Test of impairment



# Newer approach to evaluation

- Frequency:
  - Binomial method
  - Test $p<0.1$
- Magnitude
  - Acceptance sampling by variables
  - Tolerance interval on percentile
  - Test criteria by computing mean for the distribution of measurements and comparing with what is expected given the percentile criteria

# Problems

- Approach is local
  - Limited sampling budget; many stations means small sample sizes per station
  - Impairment may occur over a region
  - Modeling must be relatively simple (hard to account for seasonality, temporal effects)
  - Does not complement current approaches to sampling
  - Site history is ignored
  - Not linked to TMDL analysis (regional) and 305 reporting

# Probabilistic sampling schemes

- Rotating panel surveys
  - Some sites sampled at all possible times
  - Other sites sampled on rotational basis
  - Sites in second group may be randomly selected

# Making the assessment regional

Y = mean + site
Y = mean + time + site

$$\boldsymbol{y} = X\boldsymbol{\beta} + Z\boldsymbol{u} + \boldsymbol{e}$$

- X defines fixed effects (time), Z defines random ones (site, location), β, u are parameters
- Covariances

$$\boldsymbol{e} \sim MVN(0,\Gamma)$$
$$\boldsymbol{u} \sim MVN(0,G)$$

$$V(y) = ZGZ' + \Gamma$$

# Regional Mixed Model

- Allows for covariates
- Allows for a variety of error structures
  - Temporal, spatial, both
- Does not require equal sample sizes etc
- Allows estimation of means for sites with small sample sizes
  - Improves estimation by borrowing information from other sites

# Simple model

$$y_{ij} = m + a_i + e_{ij}$$

Random site effect

Error term allows for modeling of temporal or spatial correlation

- Testing is based on estimate and variance of mean for site i ($\mu_i$)
- Can also test for regional impairment using distribution of grand mean

# Error and stochastic components

$$y_{ij} = m + a_i + e_{ij}$$

Error term allows for modeling of temporal or spatial correlation

Random site effect

- Covariance Structure without correlation (one random effect model)

$$e_{ij} \overset{iid}{\sim} N(0, s^2)$$

- Spatial Covariance Structure

$$Var(e) = \sigma^2 \begin{bmatrix} 1 & \rho^{d_{12}} & \rho^{d_{13}} & \rho^{d_{14}} \\ \rho^{d_{21}} & 1 & \rho^{d_{23}} & \rho^{d_{24}} \\ \rho^{d_{31}} & \rho^{d_{32}} & 1 & \rho^{d_{34}} \\ \rho^{d_{41}} & \rho^{d_{42}} & \rho^{d_{43}} & 1 \end{bmatrix}$$

# Test based on OLS estimations for each site i

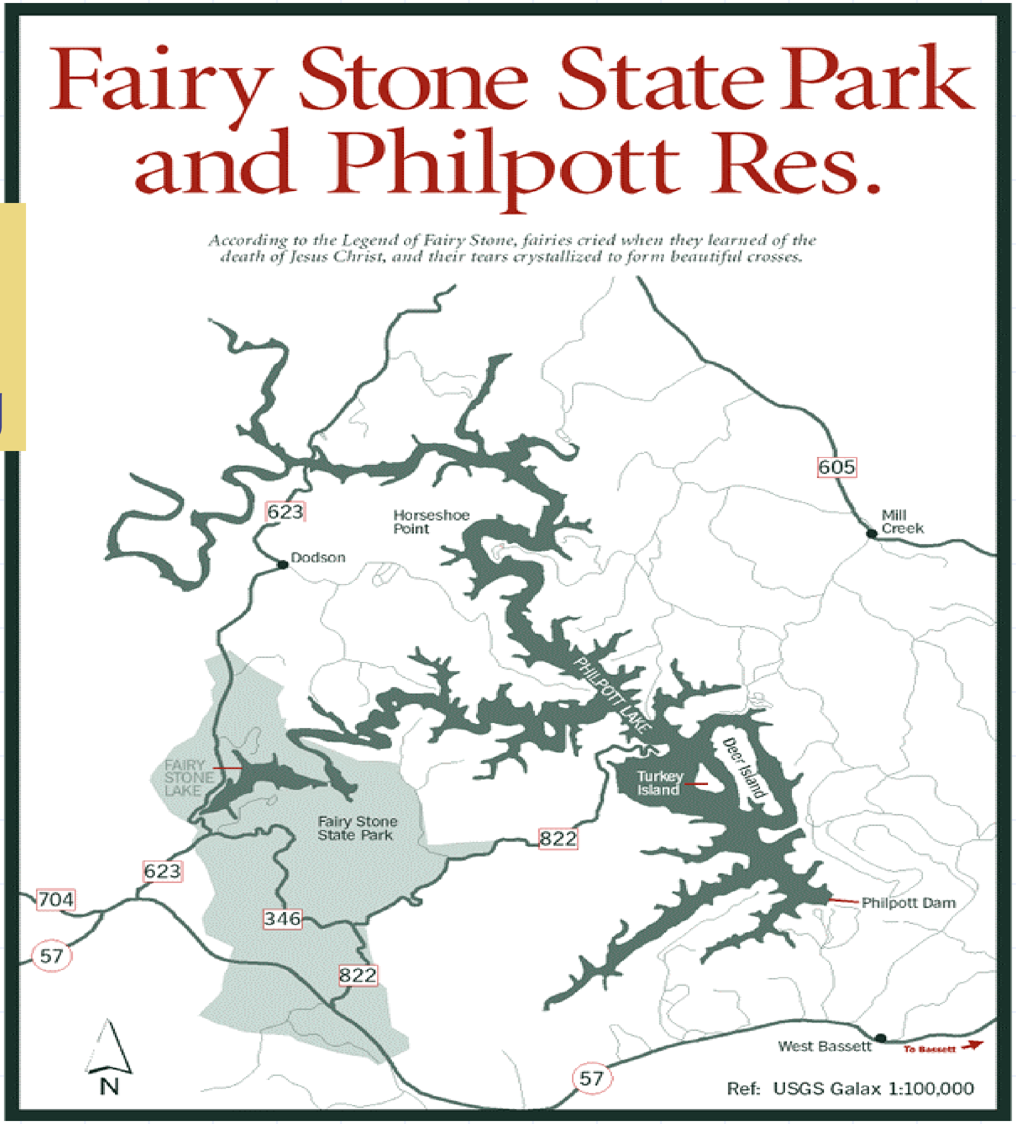
$$\frac{\bar{y}_i - baseline}{\hat{s} / \sqrt{n_i}} \sim t_{df,d}$$

where $\bar{y}_i$ and $\hat{s}$ are OLS estimates of $m$ and $s$;

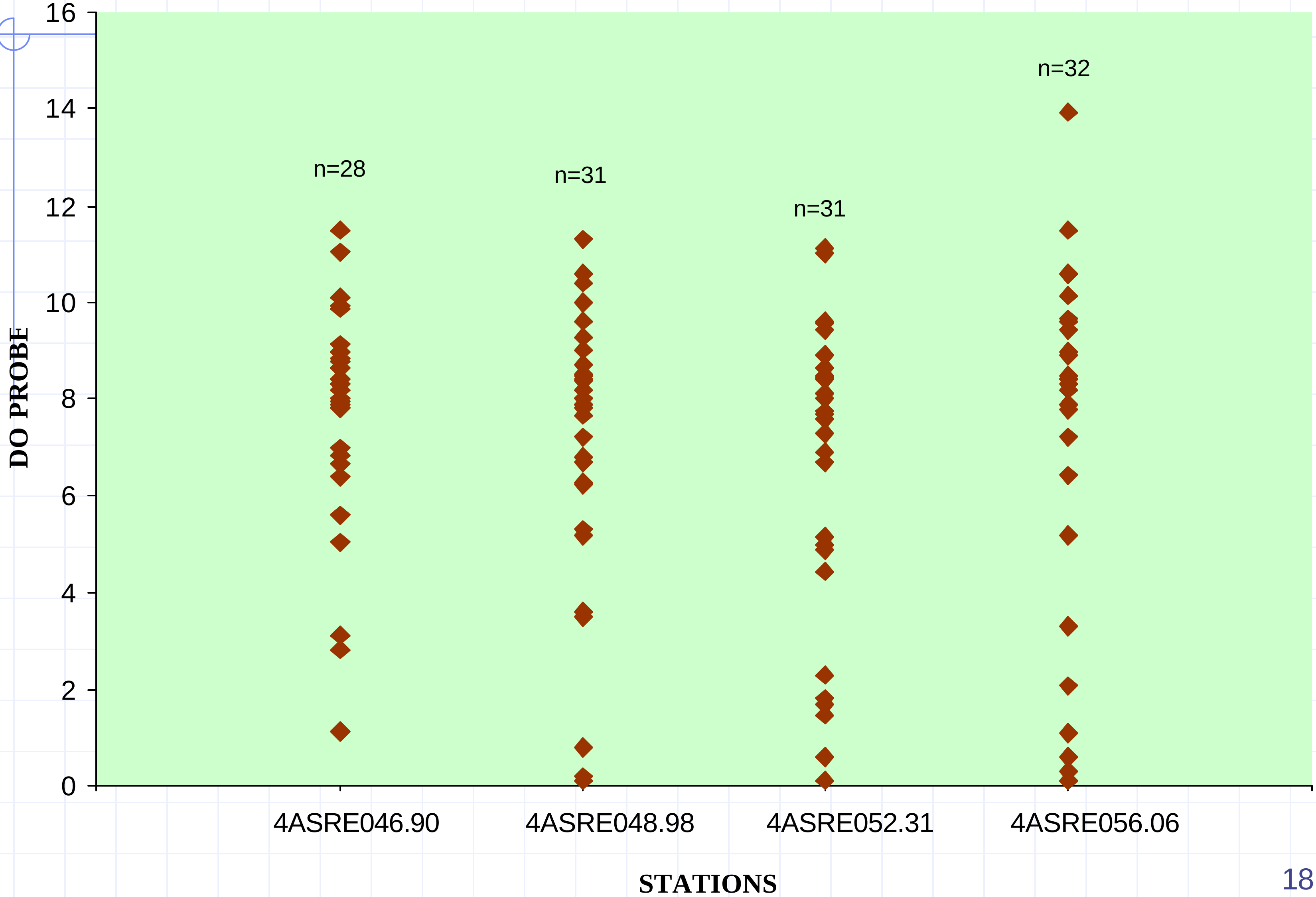
$df = n_i - 1,\ d = noncentrality$

- Baseline is the standard. For DO, we use 5, and for PH 6.
- Model based: same idea but mean and variance are estimated from model

Located in SW Virginia
Good bass fishing



# DO data collected at four stations of PHILPOTT RESERVOIR (years 2000, 2001 & 2002)



# Evaluation based on Do data of PHILPOTT RESERVIOR (2000-2002)

| | 4ASRE046.90 | Model based | 4ASRE052.31 | 4ASRE056.06 |
|---|---|---|---|---|
| n | 28 | | 31 | 32 |
| Sample mean | 7.55 | | 6.66 | 6.67 |
| Sample variance | 5.81 | | 9.56 | 16.15 |
| % excceding | 11 | | 26 | 28 |
| Binomial p-value | .5406 | | .0096 | .0033 |
| Test statistic | 5.6 | 4.27 | 2.99 | 2.35 |
| critical value | 4.75 | 5.05 | 5.19 | 5.2 |
| conclusion | **Fail to reject** | **reject** | reject | reject |

Single site analysis

# Bayesian approach

$$y_{ij} = m + a_i + e_{ij}$$

- a is a random site effect
- Error term may include temporal correlation or spatial
- Priors on parameters
  - Mean –uniform
  - a is normal (random effect) variance has prior

$$p(s^2, s_a^2) \propto \frac{1}{s^2} \frac{1}{s^2 + s_a^2}$$

# Alternative: Using historical data

- Power prior – Chen, Ibrahim, Shao 2000
- Use likelihood from the previous assessment ($D_0$). Basic idea: weight new data by prior data
- Power term, $d$ , determines influence of historical data.
- Modification to work with Winbugs

# Incorporate Historical Data using Power Priors

- Make $d$ random, and assign a prior $p(d) = Beta(a, b)$ on it. The joint posterior of $(q, d)$ becomes

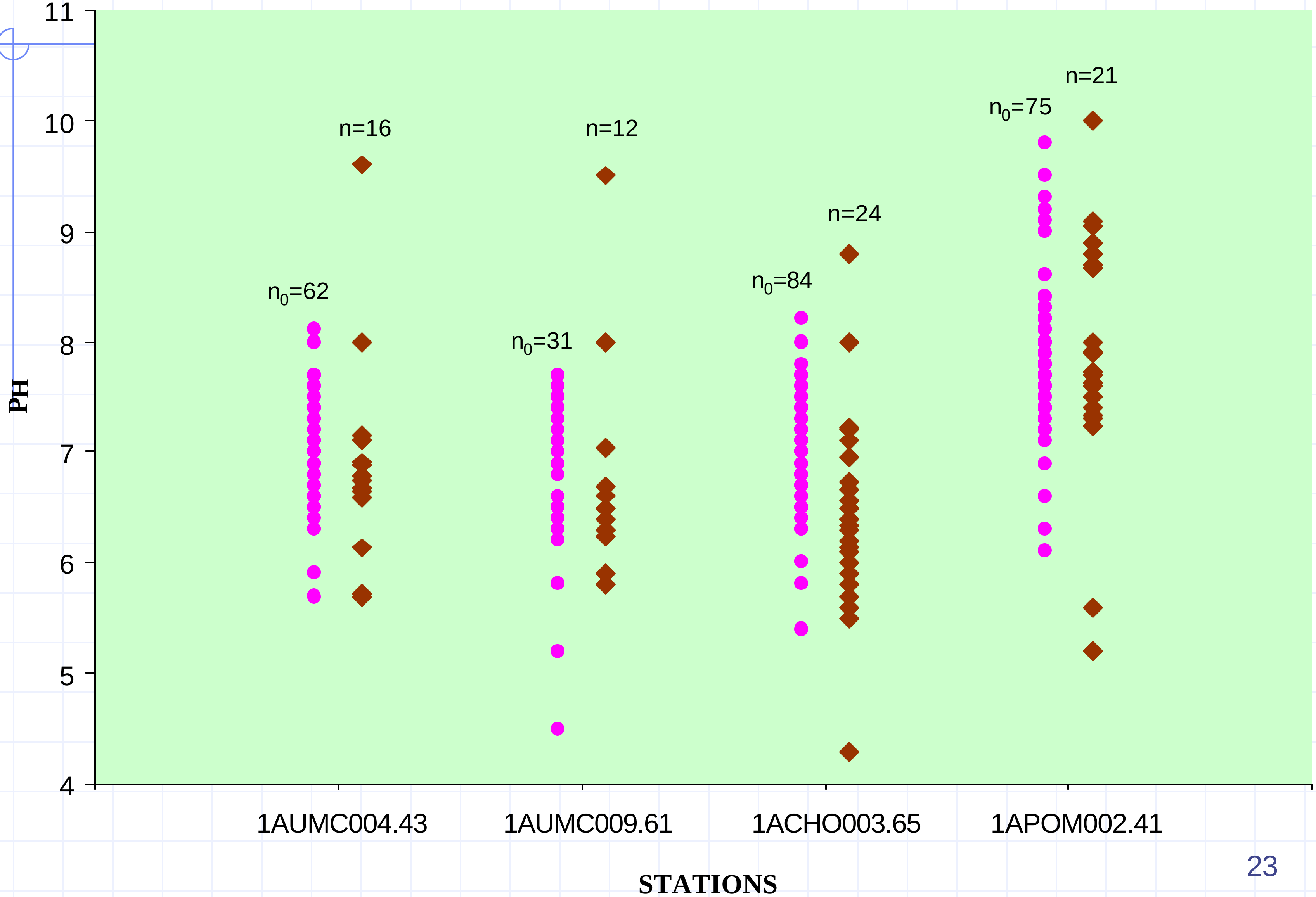
$$p(q, d \mid D_0, D) \propto \frac{L(q \mid D)(L(q \mid D_0))^d p(q) p(d)}{\int (L(q \mid D_0))^d p(q) dq} I_A(d)$$

where D is current data and $D_0$ is past data

$$A = \left\{ d : 0 < \int p(q) \left( L(q \mid D_0) \right)^d dq < \infty \right\}$$

- Advantage: Improve the precision of estimates.

# PH data collected at four stations: use past information to build prior



# Evaluate site impairment based on PH data with power priors

| Station of interest | 1AUMC004.43 | 1AUMC009.61 | 1Acho003.65 | 1APOM002.41 |
|---|---|---|---|---|
| n | 16 (yr.99-02) | 12 (yr.99-01) | 24 (yr.99-01) | 21 (yr.99-00) |
| No. obs <6 | 2 | 2 | 6 | 2 |
| sample mean | 6.91 | 6.78 | 6.43 | 7.87 |
| sample variance | 0.82 | 1.06 | 0.78 | 1.23 |
| $n_0$ | 62 (yr.90-98) | 31 (yr.90-98) | 84 (yr.90-98) | 75 (yr.90-98) |
| sample mean of $D_0$ | 7.05 | 6.73 | 6.95 | 7.88 |
| Percent exceed the EPA standard | 0.13 | 0.17 | 0.25 | 0.10 |
| P-value of Binomial test ($H_0$: p=0.1 $H_a$: p>0.1) | 0.4853 | 0.3410 | 0.0277 | 0.6353 |
| Bayesian test. ($H_0$: L=6 $H_a$: L<6), L is the lower 10th percentile of PH | | | | |
| With Reference Prior: | | | | |
| $P(H_0)$ | 0.1663 | **0.0502** | 0.0003 | 0.8673 |
| posterior s.d. of ? | 0.3399 | 0.4708 | 0.262 | 0.3564 |
| With Power Prior: | | | | |
| $P(H_0)$ | 0.4868 | **0.03525** | 0.0017 | 0.9831 |
| posterior s.d. of ? | 0.2566 | 0.2562 | 0.2381 | 0.2477 |

# Power Priors with Multiple Historical Data Sets

- If multiple historical data sets are available, assign a different $d_j$ for each historical data set.

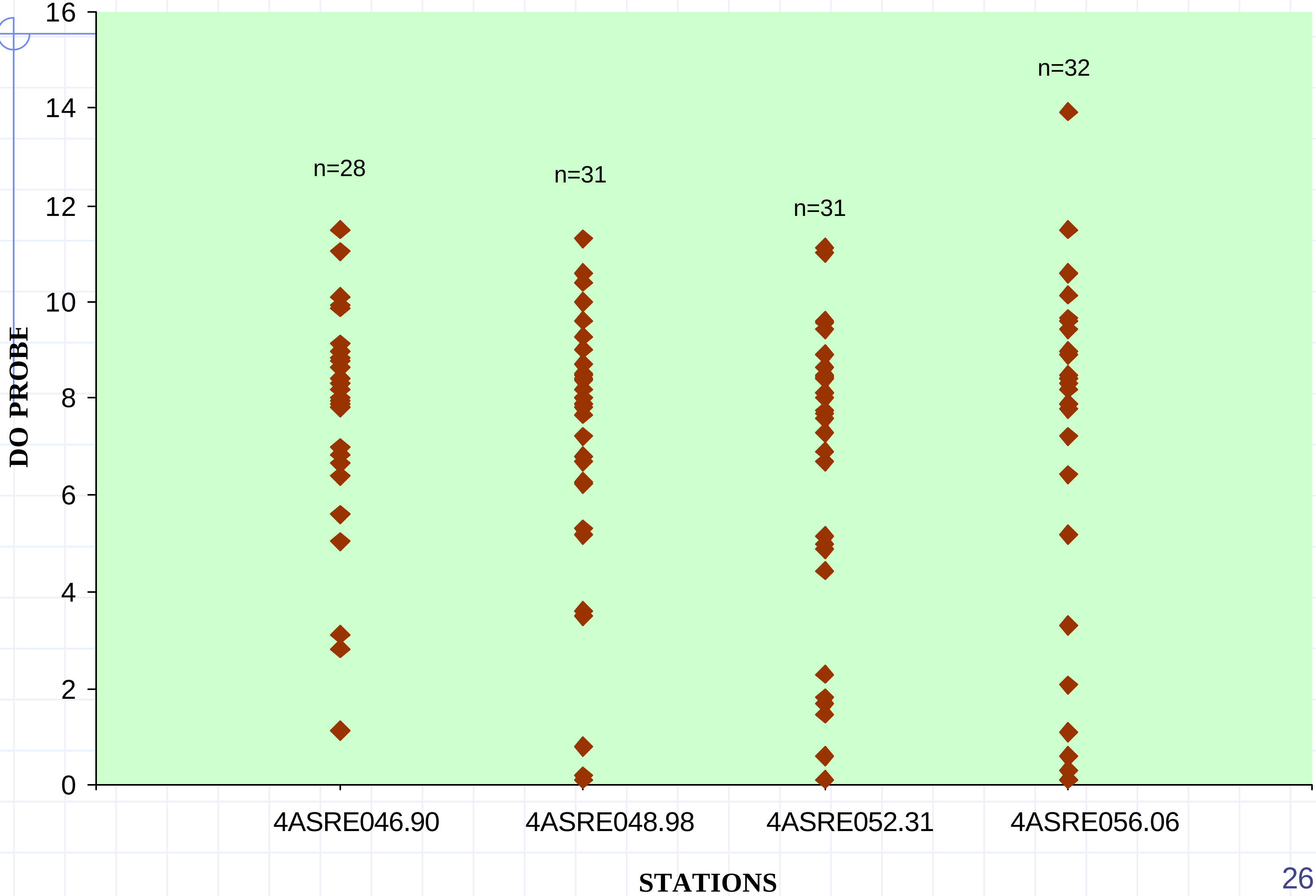
$$p(q, \underline{d} \mid D_0, D) \propto \frac{L(q \mid D)\left(\prod_{j=1}^{m} (L(q \mid D_{0j}))^{d_j} p(d_j)\right) p(q)}{\int \left(\prod_{j=1}^{m} (L(q \mid D_{0j}))^{d_j}\right) p(q)\, dq} I_B(\underline{d})$$

where

$$B = \left\{ (d_1, \ldots, d_m) : 0 < \int \left( \prod_{j=1}^{m} \left( L(q \mid D_{0j}) \right)^{d_j} \right) p(q)\, dq < \infty \right\}$$

- Data collected at adjacent stations could be used as “historical” data.

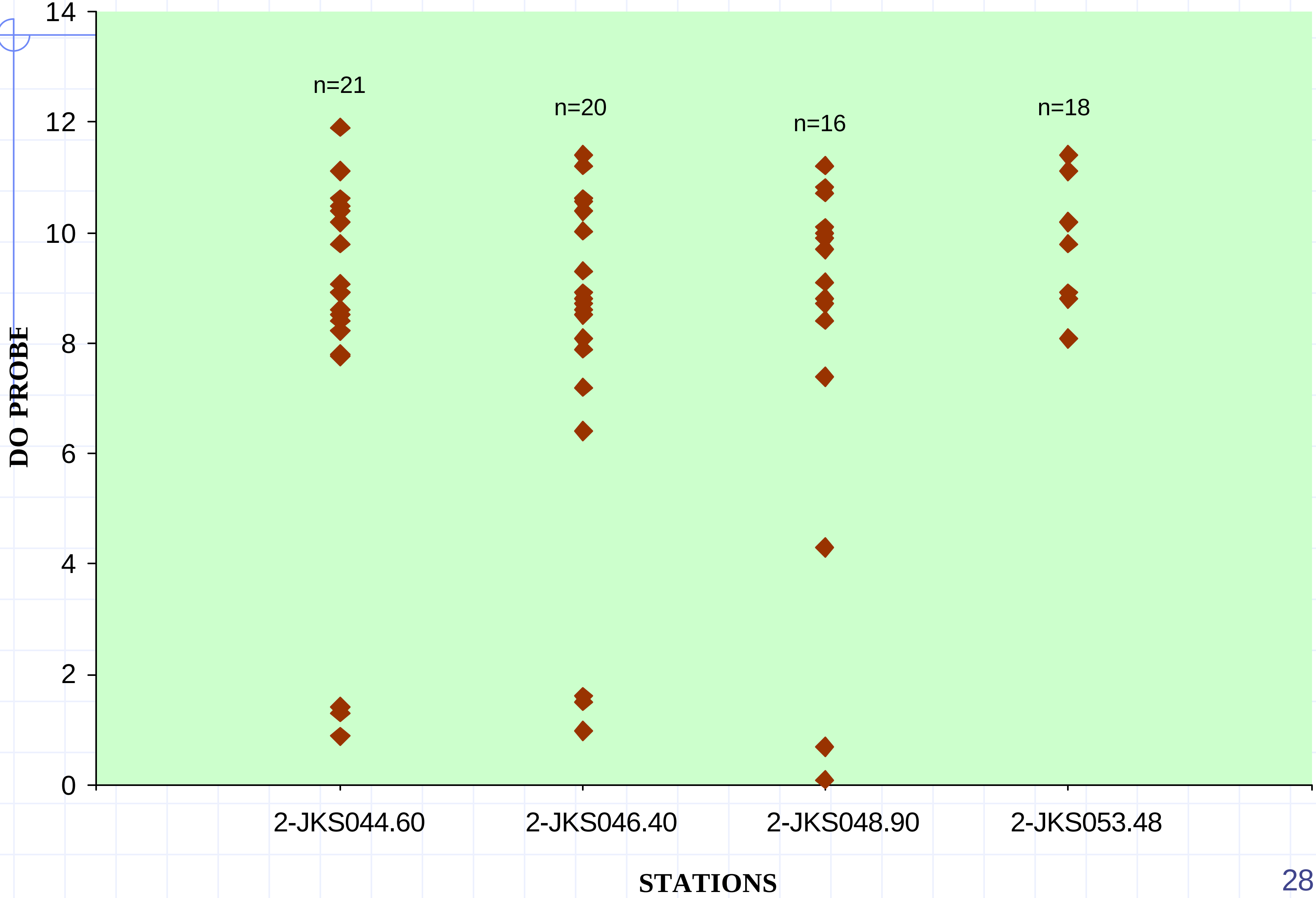
# DO data collected at four stations of PHILPOTT RESERVOIR (years 2000, 2001 & 2002)



# Evaluate site impairment based on DO data collected at four stations of PHILPOTT RESERVOIR (years 2000, 2001 & 2002)

| Station of interest | 4ASRE046.90 | 4ASRE048.98 | 4ASRE052.31 | 4ASRE056.06 |
|---|---|---|---|---|
| n | 28 | 31 | 31 | 32 |
| No. obs <5 | 3 | 5 | 8 | 9 |
| sample mean | 7.55 | 7.10 | 6.66 | 6.67 |
| sample variance | 5.81 | 8.28 | 9.56 | 16.15 |
| Percent exceed the EPA standard | 0.11 | 0.16 | 0.26 | 0.28 |
| P-value of Binomial test ($H_0$: p=0.1 $H_a$: p>0.1) | 0.5406 | 0.1932 | 0.0096 | 0.0033 |
| Bayesian test. ($H_0$: L=5 $H_a$: L<5), L is the lower 10th percentile of DO | | | | |
| With Reference Prior: | | | | |
| $P(H_0)$ | **0.1640** | 0.0038 | 0 | 0 |
| posterior s.d. of ? | 0.6514 | 0.7325 | 0.7875 | 1.008 |
| With Power Prior: | | | | |
| $P(H_0)$ | **0** | 0 | 0 | 0 |
| posterior s.d. of ? | 0.5485 | 0.5371 | 0.5439 | 0.6162 |

# DO data collected at four stations of MOOMAW RESERVOIR (years 2000 & 2001)



# Evaluate site impairment based on DO data collected at four stations of MOOMAW RESERVOIR (years 2000 & 2001)

| Station of interest | 2-JKS044.60 | 2-JKS046.40 | 2-JKS048.90 | 2-JKS053.48 |
|---|---|---|---|---|
| n | 21 | 20 | 16 | 8 |
| No. obs <5 | 3 | 3 | 3 | 0 |
| sample mean | 8.16 | 8.06 | 8.19 | 9.81 |
| sample variance | 9.73 | 10.07 | 12.14 | 1.32 |
| Percent exceed the EPA standard | 0.14 | 0.15 | 0.19 | 0.00 |
| P-value of Binomial test ($H_0$: p=0.1 $H_a$: p>0.1) | 0.3516 | 0.3231 | 0.2108 | 1.0000 |
| Bayesian test. ($H_0$: L=5 $H_a$: L<5), L is the lower 10th percentile of DO | | | | |
| With Reference Prior: | | | | |
| $P(H_0)$ | 0.1497 | 0.1149 | 0.1022 | 0.9968 |
| posterior s.d. of ? | 1.0030 | 1.0500 | 1.3110 | 0.7219 |
| With Power Prior: | | | | |
| $P(H_0)$ | 0.1338 | 0.1206 | 0.1163 | 0.3301 |
| posterior s.d. of ? | 0.6698 | 0.6832 | 0.7132 | 0.7469 |

# Comments

- Advantages
  - Greater flexibility in modeling
  - Allows for site history to be included
  - Can include spatial and temporal components
  - Can better connect to TMDL analysis and probabilistic sampling
- Disadvantage
  - Requires more commitment to the modeling process
  - Greater emphasis on the distributional assumptions